1 Numerical Exercises

1. Compute the distance for each of the following vectors a, b, c with vector d according to the following similarity measures:

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad c = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} \qquad d = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

   (a) Cross-correlation
   Solution:
   $d_{cross}(a, d) = 18 \qquad d_{cross}(b, d) = 45 \qquad d_{cross}(c, d) = 45$

   (b) Normalized Cross Correlation (NCC)
   Solution:
   $d_{ncross}(a, d) = \frac{18}{\sqrt{14}\sqrt{27}} \qquad d_{ncross}(b, d) = \frac{45}{\sqrt{77}\sqrt{27}} \qquad d_{ncross}(c, d) = \frac{45}{\sqrt{77}\sqrt{27}}$

   (c) Sum of Squared Differences (SDD)
   Solution:
   $d_{SSD}(a, d) = 5 \qquad d_{SSD}(b, d) = 14 \qquad d_{SSD}(c, d) = 14$

   (d) Sum of Absolute Differences (SAD)
   Solution:
   $d_{SAD}(a, d) = 3 \qquad d_{SAD}(b, d) = 6 \qquad d_{SAD}(c, d) = 6$

2. Compute the census transform for the the following patches A and B and the hamming distance between the two transforms.

$$A = \begin{bmatrix} 2 & 3 & 7 \\ 0 & 5 & 6 \\ 1 & 9 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 & 4 \\ 2 & 4 & 3 \\ 5 & 0 & 6 \end{bmatrix}$$

   Solution:
   We use the ordering convention shown in the lecture slides, which is counter-clockwise:

$$CT(A) = [0, 0, 1, 0, 1, 1, 0, 0] \qquad CT(B) = [0, 1, 0, 1, 0, 1, 1, 0]$$
$$d_{SAD}(CT(A), CT(B)) = 0 + 1 + 1 + 1 + 1 + 0 + 1 + 0 = 5$$

3. Compute the cornerness measure for the Moravec Corner detector for the following patch C. To avoid border artifacts, you can neglect padding.

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

   Solution:
   Since the original patch C has a patch size of 4x4, the shifted window of the Moravec Corner detector should have a size of 2x2 in order to compute the SSD for each of the eight directions. It is not possible to compute the cornerness measure with a window size of 3x3 without interpolating or padding.
   In a first step, the sums of SSDs for each direction, i.e., horizontal, vertical, diagonal 1 (left top - right bottom), diagonal 2 (left bottom - right top)) are computed for the 2x2 window in the center of patch C.

$$SSD_{horiz} = 4 + 4 = 8 \qquad SSD_{vert} = 34 + 34 = 68$$

$$SSD_{diag1} = 54 + 54 = 108 \qquad SSD_{diag2} = 22 + 22 = 44$$

The final cornerness measure corresponds to the minimum of those sums

$$R_{Moravec}(C) = min(SSD_{horiz}, SSD_{vert}, SSD_{diag1}, SSD_{diag2}) = 8$$

4. Compute the second moment matrix for the following patch D. To avoid border artifacts, you can neglect padding.

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 4 & 4 & 4 & 5 \\ 5 & 5 & 5 & 6 \end{bmatrix}$$

Solution:
In a first step, we compute the image gradients I_x and I_y in x and y direction using Sobel filters

$$I_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} * I, \qquad I_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} * I$$

$$I_x = \begin{bmatrix} 0 & 9 \\ 0 & 7 \end{bmatrix} \qquad I_y = \begin{bmatrix} 16 & 17 \\ 20 & 17 \end{bmatrix}$$

Next, we can compute the products I_x^2, I_y^2 and $I_x I_y$, which are necessary for the second order matrix

$$I_x^2 = \begin{bmatrix} 0 & 81 \\ 0 & 49 \end{bmatrix} \qquad I_y^2 = \begin{bmatrix} 256 & 289 \\ 400 & 289 \end{bmatrix} \qquad I_x I_y = \begin{bmatrix} 0 & 153 \\ 0 & 119 \end{bmatrix}$$

Based on the size of the resulting patch containing the valid gradients, we consider a window size of 2x2 for computing the second moment matrix, which corresponds to the center location of patch D. Keep in mind that the center of a patch with an even length in horizontal and vertical is not well defined.
By using the definition of the second moment matrix, we obtain the following matrix as a solution for the center pixel of patch D

$$M(u, v) = \begin{bmatrix} \sum I_x^2(u, v) & \sum I_x(u, v) I_y(u, v) \\ \sum I_x(u, v) I_y(u, v) & \sum I_y^2(u, v) \end{bmatrix},$$

$$M = \begin{bmatrix} 130 & 272 \\ 272 & 1234 \end{bmatrix}$$

5. Which of the following matrices correspond to the second moment matrix M approximating the sum of squared differences for the given patch A?
Assume the following conditions $\lambda_1 \neq 0, \lambda_2 \neq 0$, $M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$

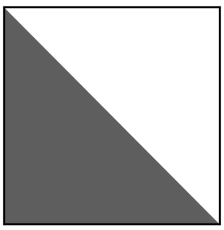


Figure 1: Patch A

(a) $M = \begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}$

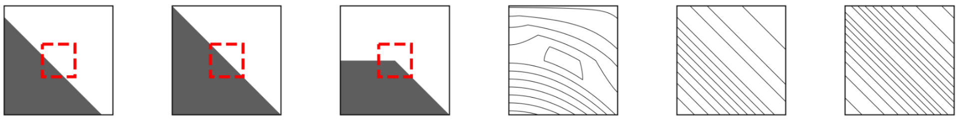
(b) $M = \begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}$

(c) $M = \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix}$

(d) $M = \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix}$

Solution: (b)

6. Consider the image patches within the red dashed squares. Which image patch [A, B, C] corresponds to which SSD cost landscape [1, 2, 3]? Curves in the SSD landscapes [1, 2, 3] indicate points of constant SSD values, i.e., all points on one curve have the same SSD cost.



A B C 1 2 3

(a) A - 3, B - 2, C - 1
(b) A - 3, B - 1, C - 2
(c) A - 2, B - 3, C - 1
(d) A - 2, B - 1, C - 3

Solution: (c)

7. Discuss the impact of image resolution on Harris corner detection. How might this affect the performance of a vision-based robot operating in environments with varying distances to objects?

Solution:

Optimal resolution depends on the specific robotic application and environment. Adaptive techniques are often necessary to balance performance across varying distances.

- Higher resolution: detects more corners, improves corner localization precision, but increases computation time (and power consumption)
- Near objects might have many corners, while far ones fewer, leading to inconsistent detections missing important features
- Ideally find balance between detail and processing speed, e.g. with adaptive resolution based on the task or combining with other sensors

8. A robotics company is developing a vision system for a warehouse robot that needs to identify and navigate around various objects. They decide to use Harris corner detection as part of their object recognition pipeline. What are two potential challenges they might face when implementing this algorithm in a warehouse environment, and how could they address each challenge?

Solution:

- Varying lighting: Use adaptive thresholding based on local image statistics.
- Repetitive patterns: Implement post-processing to filter and cluster corners, combine with other feature descriptors.

9. A self-driving car manufacturer wants to incorporate Harris corner detection in their vehicle's computer vision system for lane detection and obstacle avoidance. How might the choice of parameters for the Harris corner detector affect the system's performance in various driving scenarios (e.g., urban vs. highway, day vs. night)? Suggest two specific parameter adjustments and explain their potential impacts.

   Solution:

   - Sensitivity threshold (k):
     - Higher k: Reduces noise in complex environments
     - Lower k: Increases sensitivity for subtle features
   - Neighborhood size:
     - Smaller: Better for fine details in urban areas
     - Larger: Improved detection of large-scale features on highways, for example at night with low light
   - Additional option: Implement dynamic parameter adjustment based on driving conditions