1 Numerical Exercises

1. Consider the clustering of the following points in $\mathbb{R}^2$ using the $k$-means clustering, where $k = 2$.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>0 0</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0 1</td>
</tr>
<tr>
<td>$x_3$</td>
<td>-1 2</td>
</tr>
<tr>
<td>$x_4$</td>
<td>2 0</td>
</tr>
<tr>
<td>$x_5$</td>
<td>3 0</td>
</tr>
<tr>
<td>$x_6$</td>
<td>4 -1</td>
</tr>
</tbody>
</table>

Table 1: Datapoints

(a) In a first step, compute the squared distance matrix $D_{ij} = \text{dist}_{\text{eucl}}(\mu_i, x_j)^2$ between the datapoints $x_j$ and the initial cluster centers $\mu_i$. Assume that the first and last datapoint are the initial centers.

(b) Based on the distance matrix $D_{ij}$, perform one iteration (cluster assignment and center update) of the k-mean clustering algorithm. Solution

Based on the distance matrix $D_{ij}$, the two cluster centers $\mu_1$ and $\mu_2$ have the following assignment $P_1 = \{x_1, x_2, x_3, x_4\}$ and $P_2 = \{x_5, x_6\}$.

The updated cluster centers $\mu'_1$ and $\mu'_2$ are computed by taking the mean of the corresponding assignment.

$$
\mu'_1 = \frac{1}{|P_1|} \sum_{x \in P_1} x = \frac{1}{4} \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) = \frac{1}{4} \begin{bmatrix} 1 \\ 3 \end{bmatrix}
$$

$$
\mu'_2 = \frac{1}{|P_2|} \sum_{x \in P_2} x = \frac{1}{2} \left( \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 7 \\ -1 \end{bmatrix}
$$

2. Consider the following query image with the set of visual words $Q$ and the image vocabulary $V$. Using the image retrieval method presented in the lecture, construct the voting array and state which image ($A$, $B$, $C$ or $D$) is the closest to the query image.

$$Q = \{1, 2, 3, 4\}$$

$$V = \{1 = \{A, B\}, 2 = \{A, B, C\}, 3 = \{C\}, 4 = \{A, B, C, D\}\}$$
3. Consider the following MLP with the black numbers above the edges representing the weights and the blue numbers above the arrows the biases. All activations are ReLU function, i.e., $f(x) = \max(0, x)$. Compute the hidden activations $h_1$ and $h_2$ and output $y_1$ for the following inputs to the network.

(a) $x_1 = 0$ and $x_2 = 0$

(b) $x_1 = 1$ and $x_2 = 0$

(c) $x_1 = 0$ and $x_2 = 1$

(d) $x_1 = 1$ and $x_2 = 1$

(e) For the above binary inputs, what function does this MLP approximate?