Place Recognition / Deep Learning

1 Numerical Exercises

1. Consider the clustering of the following points in $\mathbb{R}^2$ using the k-means clustering, where $k = 2$.

x_1	0	0
x_2	0	1
x_3	-1	2
x_4	2	0
x_5	3	0
x_6	4	-1

Table 1: Datapoints

   (a) In a first step, compute the squared distance matrix $D_{ij} = dist_{eucl.}(\mu_i, x_j)^2$ between the datatpoints x_j and the initial cluster centers μ_i. Assume that the first and last datapoint are the initial centers.

   (b) Based on the distance matrix D_{ij}, perform one iteration (cluster assignment and center update) of the k-mean clustering algorithm. **Solution**
   Based on the distance matrix D_{ij}, the two cluster centers μ_1 and μ_2 have the following assignment $P_1 = \{x_1, x_2, x_3, x_4\}$ and $P_2 = \{x_5, x_6\}$.
   The updated cluster centers μ_1' and μ_2' are computed by taking the mean of the corresponding assignment.

$$\mu_1' = \frac{1}{|P_1|} \sum_{x \in P_1} = \frac{1}{4}(\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}) = \frac{1}{4} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

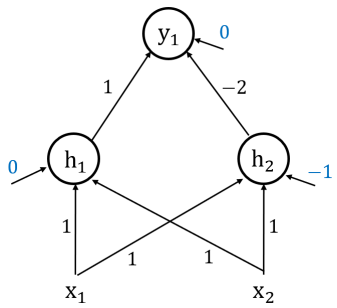
$$\mu_2' = \frac{1}{|P_2|} \sum_{x \in P_2} = \frac{1}{2}(\begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ -1 \end{bmatrix}) = \frac{1}{2} \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

2. Consider the following query image with the set of visual words Q and the image vocabulary V. Using the image retrival method presented in the lecture, construct the voting array and state which image (A, B, C or D) is the closest to the query image.

$$Q = \{1, 2, 3, 4\}$$

$$V = \{1 = \{A, B\},\ 2 = \{A, B, C\},\ 3 = \{C\},\ 4 = \{A, B, C, D\}\}$$

3. Consider the following MLP with the black numbers above the edges representing the weights and the blue numbers above the arrows the biases. All activations are ReLU function, i.e., $f(x) = max(0, x)$. Compute the hidden activations $h_1$and h_2 and output y_1 for the following inputs to the network.



(a) $x_1 = 0$ and $x_2 = 0$

(b) $x_1 = 1$ and $x_2 = 0$

(c) $x_1 = 0$ and $x_2 = 1$

(d) $x_1 = 1$ and $x_2 = 1$

(e) For the above binary inputs, what function does this MLP aproximate?