Filtering & Edge detection

1 Numerical Exercises

1. Consider the following 1D image pixel arrays A and B

\[ A = [3, 1, 2, 1] \quad B = [7, 7, 6, 4] \]

(a) Compute the filter F, which was applied as convolution to the pixel array A resulting in the output B. Assume that zero padding was applied.

\[ B = A * F \]

(b) Compute the filter F, which was applied as cross-correlation to the pixel array A resulting in the output B. Assume that zero padding was applied.

\[ B = A \otimes F \]

(c) Compute the convolution signal C between the pixel array A and B. Use “reflect across edge” padding. The output should have the same size as the input signal.

2. What is the convolution output if filter F is applied to the following A matrix assuming zero padding?

\[
F = \begin{bmatrix}
0 & 0 & -1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
8 & 7 & 6 & 5 \\
4 & 3 & 2 & 1
\end{bmatrix}
\]

3. Find the two 1D separable filters \( a, b \in \mathbb{R}^{3 \times 1} \) resulting in the following 2D filters such that:

\[ A = a b^T \]

(a) \[ A = \begin{bmatrix}
0 & 0 & 0 \\
2 & 2 & 2 \\
0 & 0 & 0
\end{bmatrix} \]

(b) \[ A = \begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \]
4. In the lecture you’ve seen the Prewitt filter which calculates a partial derivative. For example, the filter
\[ G_x = [-1 \quad 0 \quad 1] \]
calculates the first partial derivative in the x direction. It approximates \( \frac{\partial I}{\partial x} \). In a similar fashion, derive a filter \( G_{xx} \) which approximates the second order partial derivative in x \( \frac{\partial^2 I}{\partial x^2} \). The filter should be of size 1x3.

(a) Derive the filter \( G_{xx,c} \) using central differences (i.e. one pixel on either side of the current pixel)

(b) Derive the filter \( G_{xx,f} \) using forward differences (i.e. using only information to the right side of the current pixel). The filter size may be larger than 1x3.