Filtering & Edge detection

1 Numerical Exercises

1. Consider the following 1D image pixel arrays A and B

$$A = [3, 1, 2, 1] \qquad\qquad B = [7, 7, 6, 4]$$

   (a) Compute the filter F, which was applied as **convolution** to the pixel array A resulting in the output B. Assume that zero padding was applied.

$$B = A{*}F$$

   (b) Compute the filter F, which was applied as **cross-correlation** to the pixel array A resulting in the output B. Assume that zero padding was applied.

$$B = A{\otimes}F$$

   (c) Compute the convolution signal C between the pixel array A and B. Use “reflect across edge” padding. The output should have the same size as the input signal.

2. What is the convolution output if filter F is applied to the following A matrix assuming zero padding?

$$F = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad\qquad A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

3. Find the two 1D separable filters $a, b \in R^{3\times 1}$ resulting in the following 2D filters such that:

$$A = ab^T$$

   (a) $A = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}$

4. In the lecture you've seen the Prewitt filter which calculates a partial derivative. For example, the filter

$$G_x = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

calculates the first partial derivative in the x direction. It approximates $\frac{\partial I}{\partial x}$. In a similar fashion, derive a filter G_{xx} which approximates the second order partial derivative in x $\frac{\partial^2 I}{\partial x^2}$. The filter should be of size 1x3.

(a) Derive the filter $G_{xx,c}$ using central differences (i.e. one pixel on either side of the current pixel)

(b) Derive the filter $G_{xx,f}$ using forward differences (i.e. using only information to the right side of the current pixel). The filter size may be larger than 1x3.