Multiple View Geometry 4

1 Numerical Exercises

1. In addition to computing the camera pose with 3D-2D correspondences, it is possible to estimate the relative pose transformation based on 3D-3D correspondences. In the following, you will derive the first steps for the "Least-Squares Fitting of Two 3-D Point Sets" Arun et al., PAMI87. Consider two point sets $\{p_i\}$ and $\{p'_i\}$with $i = 1, 2, ..., N$ and relation $p'_i = Rp_i + T$, where R is a rotation matrix, and T a translation vector.

   (a) Write down the least-square cost, which is minimized to find R and T.
   Solution:

   $$\Sigma^2 = \sum_{i=1}^{N} \|p'_i - (Rp_i + T)\|^2$$

   (b) Write down the transformation between the centroids p and p' of the two points sets $\{p_i\}$ and $\{p'_i\}$ and express the translation vector T based p, p' and R.
   Solution:

   $$\frac{1}{N}\sum_{i=1}^{N} p'_i = R(\frac{1}{N}\sum_{i=1}^{N} p_i) + T$$
   $$p' = Rp + T$$

   Reformulating the above equation leads to:

   $$T = \frac{1}{N}\sum_{i=1}^{N} p'_i - R(\frac{1}{N}\sum_{i=1}^{N} p_i)$$
   $$T = p' - Rp$$

   (c) Using the normalized cocordinates $q_i = p_i - p$ and $q'_i = p'_i - p'$, show that the least-square cost can be written only in terms of q_i, q'_i and R.
   Solution:
   Plugging in p_i, p'_i and T from (b) into the least-square cost Σ^2 from (a), we obtain the following formula.

   $$\Sigma^2 = \sum_{i=1}^{N} \|q'_i + p' - (R(q_i + p) + p' - Rp)\|^2$$
   $$\Sigma^2 = \sum_{i=1}^{N} \|q'_i - Rq_i\|^2$$

   (d) Briefly explain in one or two sentences how you would proceed and what are the benefits of the above computations for finding R and T.
   Solution:
   Based on the cost function in (c) for the least-squares problem, we first compute the optimal rotation matrix R using SVD independently from T. Once R is found, the translation vector T can be obtained with the formula in (b).

2. Consider a stereo camera pair with a relative transformation between the left and right camera $t_{l,r}$. Additionally, we have estimates for the relative motion $t^{ll}_{k,k+1}$ between the left camera from timestep k to $k+1$, the relative motion $t^{lr}_{k+2,k}$ between the left camera at timestep $k+2$ to the right camera at timestep k, the relative motion $t^{rr}_{k+1,k+2}$ between the right camera at time step $k+1$ and $k+2$, and the relative motion $t^{ll}_{k,k+3}$ between the left camera at timestep k and $k+3$.
   Write the optimization problem for optimizing the relative motion estimates.
   Solution:

$$\{t^{ll}_{k,k+1}, t^{rr}_{k+1,k+2}, t^{lr}_{k+2,k}\} = \arg \min_{\{t^{ll}_{k,k+1}, t^{rr}_{k+1,k+2}, t^{lr}_{k+2,k}\}} \left\| t^{ll}_{k,k+1} t_{l,r} t^{rr}_{k+1,k+2} (t_{l,r})^{-1} - t_{l,r} (t^{lr}_{k+2,k})^{-1} \right\|^2$$

   Since there is only one pose estimate for timestep $k+3$, it is not possible to optimize the relative motion estimate $t^{ll}_{k,k+3}$