Filtering & Edge detection

1 Numerical Exercises

1. Consider the following 1D image pixel arrays $A$ and $B$
   \[ A = [3, 1, 2, 1] \quad B = [7, 7, 6, 4] \]
   (a) Compute the filter $F$, which was applied as \textit{convolution} to the pixel array $A$ resulting in the output $B$. Assume that zero padding was applied.
   \[ B = A * F \]
   (b) Compute the filter $F$, which was applied as \textit{cross-correlation} to the pixel array $A$ resulting in the output $B$. Assume that zero padding was applied.
   \[ B = A \otimes F \]
   (c) Compute the convolution signal $C$ between the pixel array $A$ and $B$. Use “reflect across edge” padding. The output should have the same size as the input signal.

2. What is the convolution output if filter $F$ is applied to the following $A$ matrix assuming zero padding?
   \[
   F = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}
   \]

3. Find the two 1D separable filters $a, b \in \mathbb{R}^{3 \times 1}$ resulting in the following 2D filters such that:
   \[ A = ab^T \]
   (a) $A = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
   (c) $A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}$
4. In the lecture you’ve seen the Prewitt filter which calculates a partial derivative. For example, the filter
\[ G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \]
calculates the first partial derivative in the x direction. It approximates \( \frac{\partial I}{\partial x} \). In a similar fashion, derive a filter \( G_{xx} \) which approximates the second order partial derivative in x \( \frac{\partial^2 I}{\partial x^2} \). The filter should be of size 3x3.

(a) Derive the filter \( G_{xx,c} \) using central differences (i.e. one pixel on either side of the current pixel)

(b) Derive the filter \( G_{xx,f} \) using forward differences (i.e. using only information to the right side of the current pixel)