Harris Corner Detection and Keypoint Tracking

1 Numerical Exercises

1. Compute the distance for each of the following vectors a, b, c with vector d according to the following similarity measures:

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad c = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} \qquad d = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

   (a) Cross-correlation
   Solution:
   $d_{cross}(a, d) = 18 \qquad d_{cross}(b, d) = 45 \qquad d_{cross}(c, d) = 45$

   (b) Normalized Cross Correlation (NCC)
   Solution:
   $d_{ncross}(a, d) = \frac{18}{\sqrt{14}\sqrt{27}} \qquad d_{ncross}(b, d) = \frac{45}{\sqrt{77}\sqrt{27}} \qquad d_{ncross}(c, d) = \frac{45}{\sqrt{77}\sqrt{27}}$

   (c) Sum of Squared Differences (SDD)
   Solution:
   $d_{SSD}(a, d) = 5 \qquad d_{SSD}(b, d) = 14 \qquad d_{SSD}(c, d) = 14$

   (d) Sum of Absolute Differences (SAD)
   Solution:
   $d_{SAD}(a, d) = 3 \qquad d_{SAD}(b, d) = 6 \qquad d_{SAD}(c, d) = 6$

2. Compute the census transform for the the following patches A and B and the hamming distance between the two transforms.

$$A = \begin{bmatrix} 2 & 3 & 7 \\ 0 & 5 & 6 \\ 1 & 9 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 & 4 \\ 2 & 4 & 3 \\ 5 & 0 & 6 \end{bmatrix}$$

   Solution:
   We use the ordering convention shown in the lecture slides, which is counter-clockwise:

$$CT(A) = [0, 0, 1, 0, 1, 1, 0, 0] \qquad CT(B) = [0, 1, 0, 1, 0, 1, 1, 0]$$
$$d_{SAD}(CT(A), CT(B)) = 0 + 1 + 1 + 1 + 1 + 0 + 1 + 0 = 5$$

3. Compute the cornerness measure for the Moravec Corner detector for the following patch C. To avoid border artifacts, you can neglect padding.

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

   Solution:

Since the original patch C has a patch size of 4x4, the shifted window of the Moravec Corner detector should have a size of 2x2 in order to compute the SSD for each of the eight directions. It is not possible to compute the cornerness measure with a window size of 3x3 without interpolating or padding.
In a first step, the sums of SSDs for each direction, i.e., horizontal, vertical, diagonal 1 (left top - right bottom), diagonal 2 (left bottom - right top)) are computed for the 2x2 window in the center of patch C.

$$SSD_{horiz} = 4 + 4 = 8 \qquad SSD_{vert} = 34 + 34 = 68$$

$$SSD_{diag1} = 54 + 54 = 108 \qquad SSD_{diag2} = 22 + 22 = 44$$

The final cornerness measure corresponds to the minimum of those sums

$$R_{Moravec}(C) = min(SSD_{horiz}, SSD_{vert}, SSD_{diag1}, SSD_{diag2}) = 8$$

4. Compute the second moment matrix for the following patch D. To avoid border artifacts, you can neglect padding.

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 4 & 4 & 4 & 5 \\ 5 & 5 & 5 & 6 \end{bmatrix}$$

Solution:
In a first step, we compute the image gradients I_x and I_y in x and y direction using Sobel filters

$$I_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} * I, \qquad I_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} * I$$

$$I_x = \begin{bmatrix} 0 & 9 \\ 0 & 7 \end{bmatrix} \qquad I_y = \begin{bmatrix} 16 & 17 \\ 20 & 17 \end{bmatrix}$$

Next, we can compute the products I_x^2, I_y^2 and I_xI_y, which are necessary for the second order matrix

$$I_x^2 = \begin{bmatrix} 0 & 81 \\ 0 & 49 \end{bmatrix} \qquad I_y^2 = \begin{bmatrix} 256 & 289 \\ 400 & 289 \end{bmatrix} \qquad I_xI_y = \begin{bmatrix} 0 & 153 \\ 0 & 119 \end{bmatrix}$$

Based on the size of the resulting patch containing the valid gradients, we consider a window size of 2x2 for computing the second moment matrix, which corresponds to the center location of patch D. Keep in mind that the center of a patch with an even length in horizontal and vertical is not well defined.
By using the definition of the second moment matrix, we obtain the following matrix as a solution for the center pixel of patch D

$$M(u, v) = \begin{bmatrix} \sum I_x^2(u, v) & \sum I_x(u, v)I_y(u, v) \\ \sum I_x(u, v)I_y(u, v) & \sum I_y^2(u, v) \end{bmatrix},$$

$$M = \begin{bmatrix} 130 & 272 \\ 272 & 1234 \end{bmatrix}$$

5. Which of the following matrices correspond to the second moment matrix M approximating the sum of squared differences for the given patch A?
Assume the following conditions $\lambda_1 \neq 0, \lambda_2 \neq 0,\ M = \begin{bmatrix} \sum I_x^2 & \sum I_xI_y \\ \sum I_xI_y & \sum I_y^2 \end{bmatrix}$

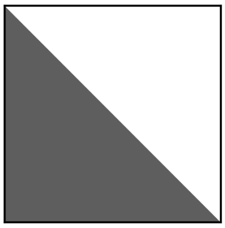


Figure 1: Patch A

(a) $M = \begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}$

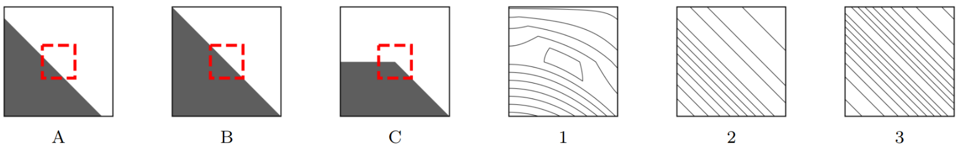
(b) $M = \begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}$

(c) $M = \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix}$

(d) $M = \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix}$

Solution: (b)

6. Consider the image patches within the red dashed squares. Which image patch [A, B, C] corresponds to which SSD cost landscape [1, 2, 3]? Curves in the SSD landscapes [1, 2, 3] indicate points of constant SSD values, i.e., all points on one curve have the same SSD cost.



(a) A - 3, B - 2, C - 1

(b) A - 3, B - 1, C - 2

(c) A - 2, B - 3, C - 1

(d) A - 2, B - 1, C - 3

Solution: (c)