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Vision Algorithms for Mobile Robotics

Lecture 10 Multiple View Geometry 4

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Next week, seminar by NASA JPL

- When: Thursday December 2nd at 8:00 am followed by Lecture 11
- Title: "Vision-Based Navigation for Mars Helicopters"
- Who: Dr. Jeff Delaune: https://www-robotics.jpl.nasa.gov/people/Jeff_Delaune/





Jet Propulsion Laboratory California Institute of Technology





Lab Exercise – Today

Intermediate VO integration for mini projects:

- problem statement
- details about what can/needs to be done
- we will show some of best examples from last years
- we will go through FAQ such as what can be added to get up to +0.5 mark

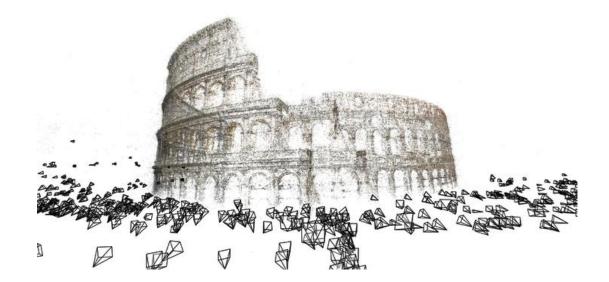
Image sequence
Feature detection
Feature matching (tracking)
Motion estimation
Local optimization

n-View Structure From Motion

- <u>Compute initial structure and motion using either:</u>
 - Hierarchical SFM
 - Sequential SFM \rightarrow Visual Odometry (VO)
- Refine simultaneously structure and motion through BA

Hierarchical SFM applied to random internet images

- Reconstruction from 150,000 images from Flickr associated with the tags "Rome"
- 4 million 3D points. Cloud of 496 computers. 21 hours of computation!

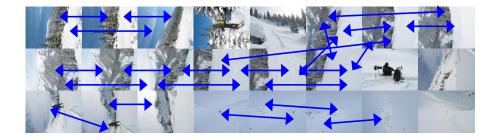


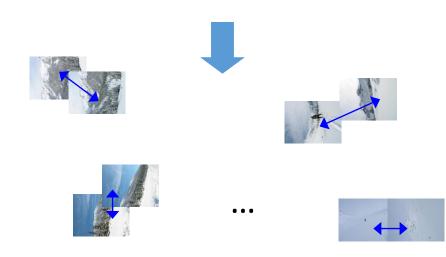
Agarwal, Snavely, Simon, Seitz, Szeliski, *Building Rome in a Day*, International Conference on Computer Vision (ICCV), 2009. <u>PDF, code, datasets</u> **Most influential paper of 2009**

State of the art software: COLMAP

Hierarchical SFM

- 1. Extract and match features between nearby frames
- 2. Build clusters consisting of 2 nearby frames

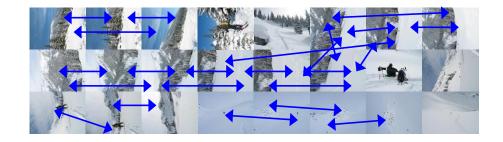


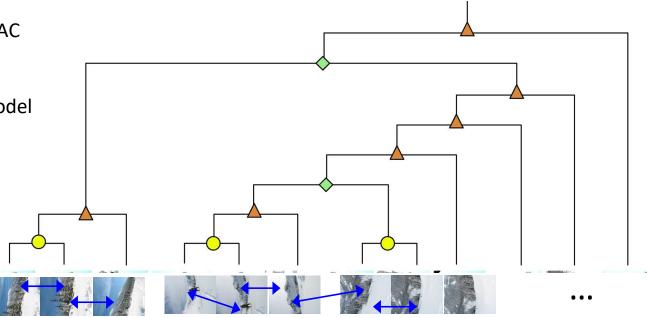


Hierarchical SFM

- 1. Extract and match features between nearby frames
- 2. Build clusters consisting of 2 nearby frames
- 3. Extract topological tree (e.g., count number of SIFT matches)
- 4. Start from the terminal nodes
 - 1. Compute 2-view SFM and build 3D model (point cloud)
- 5. Iterate according to tree structure:
 - 1. Merge new view by running 3-point RANSAC between 3D model and 3rd view
 - 2. Merge near-by models using by running again 3-point RANSAC between one 3D model and one view of the other 3D model
 - 3. Bundle adjust

The circle \circ corresponds to the creation of a stereomodel, the triangle \triangle corresponds to applying PNP, the diamond \diamond corresponds to a fusion of two partial independent models.





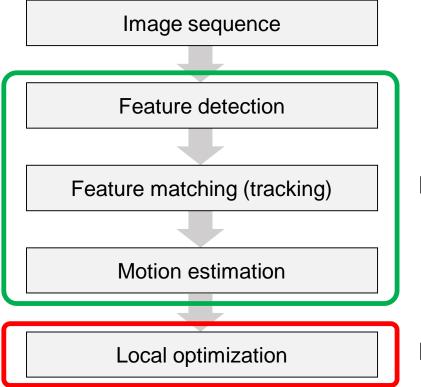
n-View Structure From Motion

- Compute initial structure and motion using either:
 - Hierarchical SFM
 - Sequential SFM \rightarrow Visual Odometry (VO)
- Refine simultaneously structure and motion through BA

Sequential SFM (also called Visual Odometry (VO))

- Initialize structure and motion from 2 views (**bootstrapping**)
- For each additional view
 - Determine pose (localization)
 - Extend structure, i.e., extract and triangulate new features (mapping)
 - Refine structure and motion through Bundle Adjustment (BA) (**optimization**)

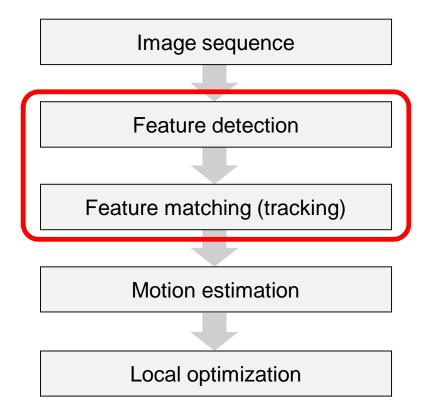
• VO computes the camera path incrementally (pose after pose)



Front-end: outputs the *relative pose* between the *last two frames*

Back-end: "adjusts" the relative poses among multiple recent frames

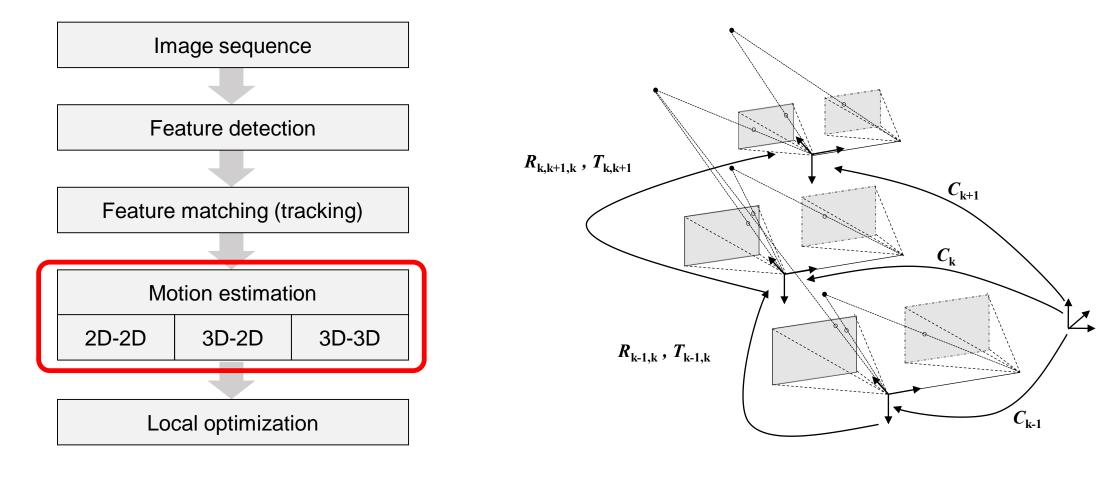
• VO computes the camera path incrementally (pose after pose)



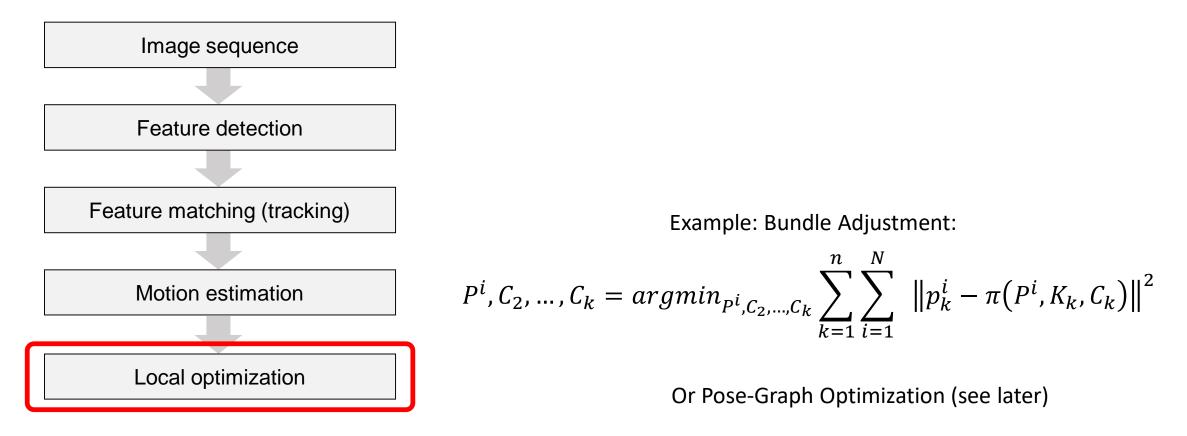


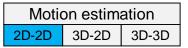
Features tracked over multiple recent frames overlaid on the last frame

• VO computes the camera path incrementally (pose after pose)



• VO computes the camera path incrementally (pose after pose)

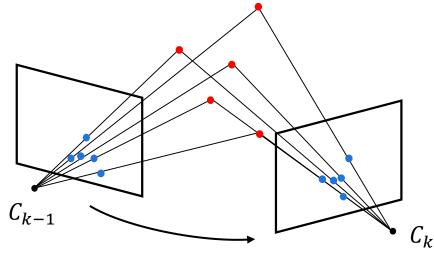




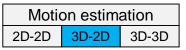
2D-to-2D (already seen: Lecture 08)

Motion from 2D-to-2D feature correspondences

- Both feature correspondences f_{k-1} and f_k are specified in image coordinates (2D)
- The minimal-case solution involves 5 feature correspondences
- Popular algorithms: **5- and 8-point algorithms**



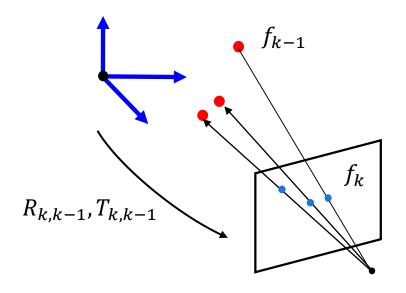
 $R_{k,k-1}, T_{k,k-1}$



3D-to-2D (already seen: Lecture 03)

Motion from 3D-to-2D feature correspondences (i.e., Perspective from *n* Points: PnP problem)

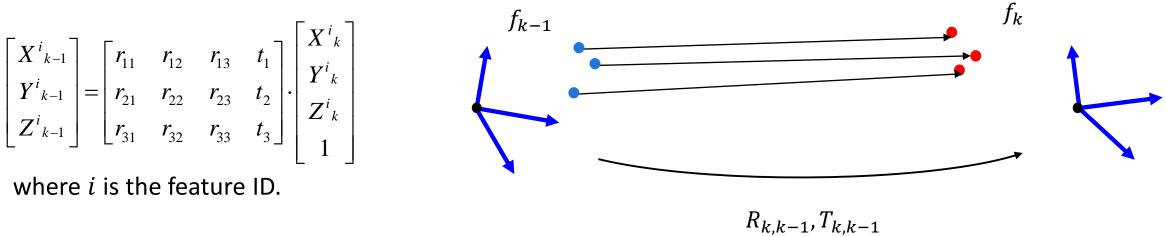
- f_{k-1} is specified in 3D and f_k in 2D
- Minimal case:
 - DLT algorithm: minimal case: 6 points from 3D objects, or 4 from planar objects
 - P3P algorithm: minimal case: 3 points (+1 for disambiguation)
 - EPNP algorithm: for more than 4 points



Motion estimation 2D-2D 3D-2D 3D-3D

3D-to-3D

- Motion from 3D-to-3D feature correspondences (also known as point cloud registration problem)
- Both f_{k-1} and f_k are specified in 3D. To do this, it is necessary to first triangulate 3D points (e.g. use a stereo camera)
- The minimal-case solution involves **3 non-collinear** correspondences
- Popular algorithm: [Arun'87]
- Consists of solving the following system of equations with R and T as unknowns:

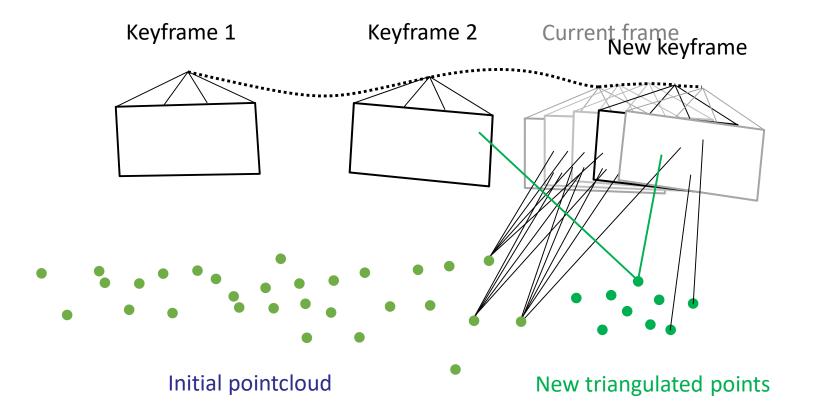


Arun, Huang, Blostein, "Least-Squares Fitting of Two 3-D Point Sets," Transactions on Pattern Analysis and Machine Intelligence (PAMI), 1987. PDF

Motion Estimation: Recap

Type of correspondences	Monocular	Stereo
2D-2D	Х	
3D-2D	Х	Х
3D-3D		Х

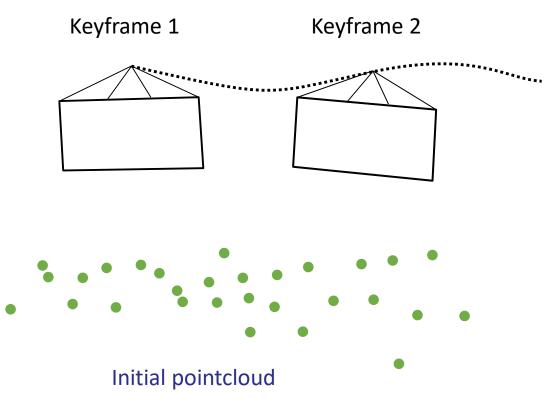
This pipeline was initially proposed in PTAM (Parallel Tracking & Mapping) [Klein, ISMAR'07]

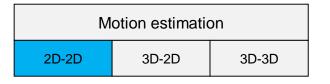


Klein, Murray, *Parallel Tracking and Mapping for Small AR Workspaces*, International Symposium on Mixed and Augmented Reality (ISMAR), 2007. <u>PDF, code, videos</u>. **Best paper award**

1. Bootstrapping (i.e., initialization)

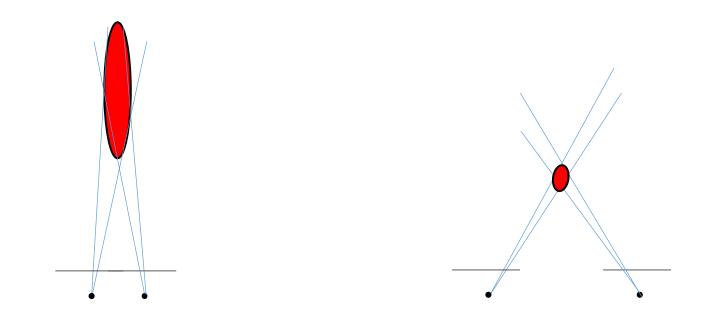
- Initialize structure and motion from 2 views: e.g., **5- or 8-point RANSAC**
- Refine structure and motion (Bundle Adjustment)
- How far should the two frames (i.e., keyframes) be?





2. Keyframe selection (i.e., skipping frames)

• When frames are taken at nearby positions compared to the scene distance, 3D points will exibit large uncertainty



2. Keyframe selection (i.e., skipping frames)

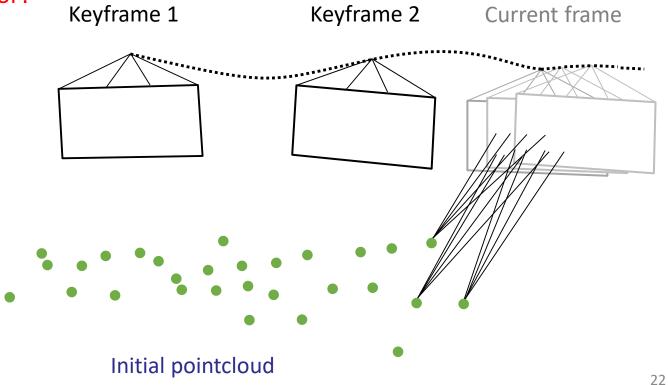
- When frames are taken at nearby positions compared to the scene distance, 3D points will exibit large uncertainty
- One way to avoid this consists of skipping frames until the average uncertainty of the 3D points, normalized by the average distance from the scene, falls below a certain threshold. The selected frames are called keyframes
- Rule of the thumb: add a keyframe when

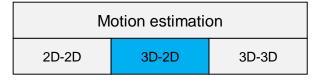
keyframe distance average-depth > threshold (~10-20 %)



3. Localization (i.e., pose estimation from a given point cloud)

- Given a 3D point cloud (map), determine the pose of each additional view
- What algorithm is used?
- How far from the last keyframe can we use it for?

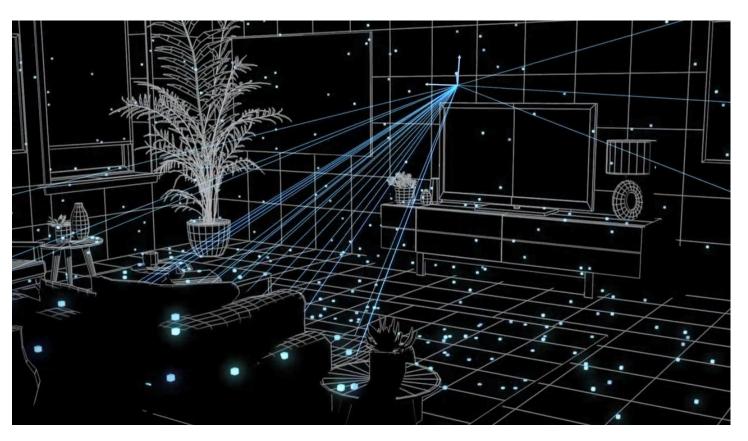




3. Localization (i.e., pose estimation from a given point cloud)

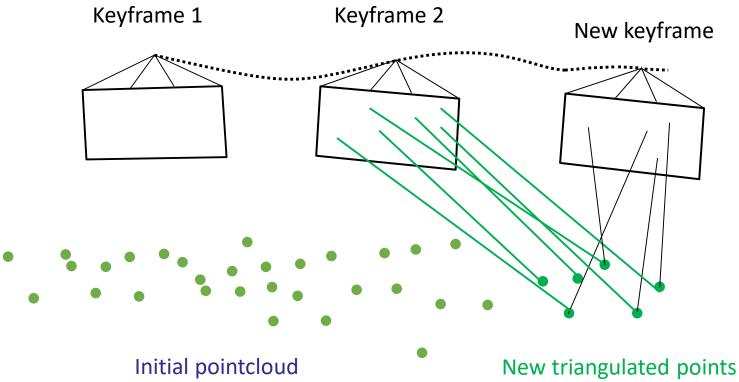
• Given a 3D point cloud (map), determine the pose of each additional view

Motion estimation		
2D-2D	3D-2D	3D-3D



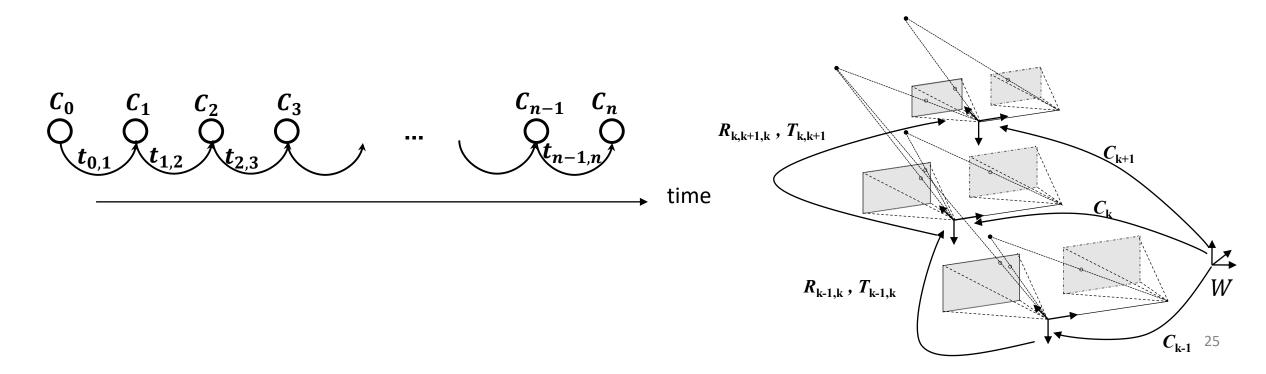
4. Extend Structure (i.e., mapping)

- Extract and triangulate new features
- Is it necessary to do this at every frame or can we just do it at keyframes?
- What are the pros and cons?



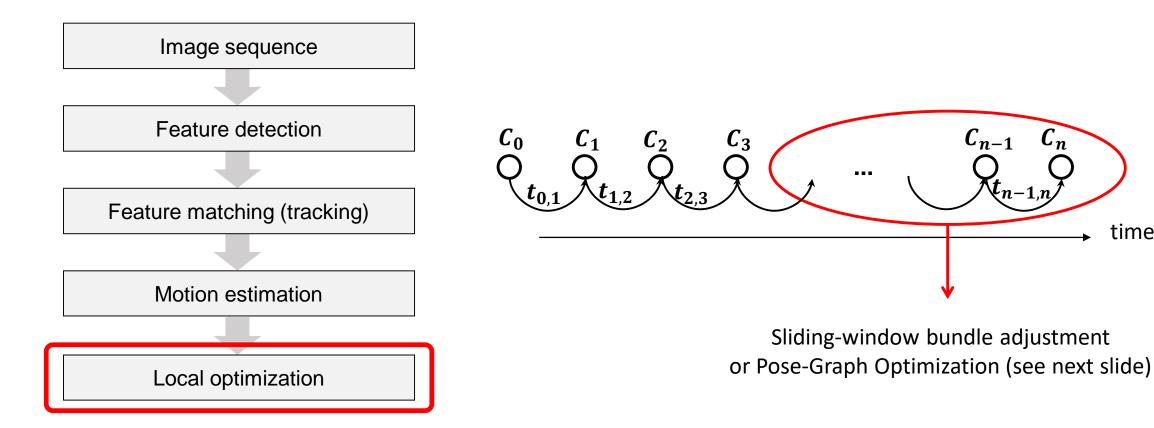
VO: putting all pieces together

- Let the **relative motion** between image I_{k-1} and image I_k be: $t_{k-1,k} = \begin{bmatrix} R_{k-1,k} & T_{k-1,k} \\ 0 & 1 \end{bmatrix}$
- Let C_{k-1} be the previous camera pose in the world reference frame
- Then, the current pose C_k in the world frame is given by: $C_k = C_{k-1}t_{k-1,k}$



Local Optimization

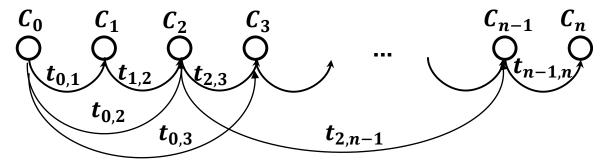
VO flowchart:



time

Pose-Graph Optimization

• So far we assumed that the transformations are between consecutive frames

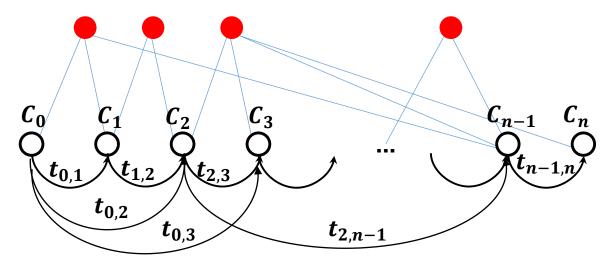


 However, transformations can also be computed between non-adjacent frames: t_{j,i} (e.g., when features from previous keyframes are still observed). They can be used as additional constraints to improve camera poses by solving:

$$\{C_{1}, ..., C_{n}\} = argmin_{\{C_{1}, ..., C_{n}\}} \sum_{i} \sum_{j} \|C_{i} - C_{j}t_{j,i}\|^{2}$$

- For efficiency, only the last *m* keyframes are used
- Gauss-Newton or Levenberg-Marquadt are typically used to minimize it. For large graphs, efficient opensource tools exist: <u>g2o</u>, <u>GTSAM</u>, <u>SLAM++</u>, <u>Google Ceres</u>

Bundle Adjustment (BA)



• Similar to pose-graph optimization but it also optimizes 3D points:

$$P^{i}, C_{1}, \dots, C_{n} = argmin_{X^{i}, C_{1}, \dots, C_{n}}, \sum_{k=1}^{n} \sum_{i=1}^{N} \rho\left(p_{k}^{i} - \pi(P^{i}, K_{k}, C_{k})\right)$$

- ρ () is the **Huber or Tukey norm**
- Gauss-Newton or Levenberg-Marquadt are typically used to minimize it. For large graphs, efficient opensource tools exist: <u>g2o</u>, <u>GTSAM</u>, <u>SLAM++</u>, <u>Google Ceres</u>

Bundle Adjustment vs Pose-graph Optimization

- BA is **more precise** than pose-graph optimization because it adds additional constraints (*landmark constraints*)
- But more costly: $O((qN + lm)^3)$ with N being the number of points, m the number of cameras poses and q and l the number of parameters for points and camera poses. Workarounds:
 - A **small window size** limits the number of parameters for the optimization and thus makes real-time bundle adjustment possible.
 - It is possible to reduce the computational complexity by just optimizing over the camera parameters and keeping the 3-D landmarks fixed, e.g., (motion-only BA)

More efficient BA algoritms have recently been developed:

[2] Demmel, Sommer, Cremers, Usenko, Square Root Bundle Adjustment for Large-Scale Reconstruction, IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), 2021. <u>Paper, Video, Code</u>.

^[1] Demmel, Schubert, Sommer, Cremers, Usenko, Square Root Marginalization for Sliding-Window Bundle Adjustment, IEEE International Conference on Computer Vision (ICCV), 2021. <u>Paper, Video, Code</u>.

Place Recognition

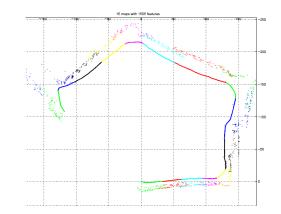
During VO, two problems can occur:

- **Relocalization problem:** camera pose estimation can fail due to:
 - 1. Feature **tracking can be lost** (due to occlusions, low texture, quick motion, illumination change)
 - 2. In case of monocular VO: pure rotation followed by translation (why?)
 - \rightarrow Solution: Re-localize camera pose and continue
- Loop closing problem
 - When you go back to a previously mapped area:
 - Loop closure detection: to avoid map duplication
 - Loop correction: to compensate the accumulated drift
 - In both cases you need a place recognition technique

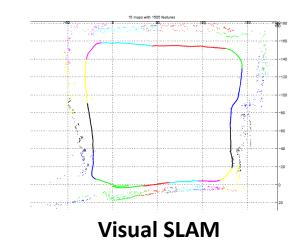
We will address place recognition in Lecture 12

VO vs. Visual SLAM (recap from Lecture 01)

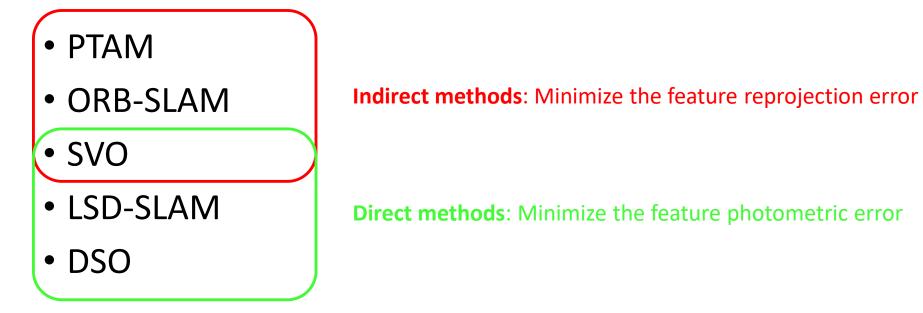
- Visual Odometry
 - Focus on incremental estimation
 - **Guarantees local consistency** (i.e., estimated trajectory is locally correct, but not globally, i.e. from the start to the end)
- Visual SLAM (Simultaneous Localization And Mapping)
 - SLAM = visual odometry + loop detection & closure
 - **Guarantees global consistency** (the estimated trajectory is globally correct, i.e. from the start to the end)



Visual odometry



Open Source Monocular VO and SLAM algorithms



PTAM: Parallel Tracking and Mapping

• Monocular only

Feature based

- FAST corners + patch descriptors
- Minimizes reprojection error
- Jointly optimizes poses & structure (sliding window BA)
- First to propose keyframe-based VO
- First to propose alternation of localization (i.e., camera tracking) and mapping
- Tracking and mapping running in two independent threads: updated map is used by localization thread asynchronously, as soon it becomes available
- Includes:
 - Relocalization
 - No global optimization, only local
- **Real-time (30Hz)**, however global optimization is not done in real time but asynchronously every once in a while

Klein, Murray, *Parallel Tracking and Mapping for Small AR Workspaces*, International Symposium on Mixed and Augmented Reality (ISMAR), 2007. <u>PDF, code, videos</u>. **Best paper award**

Parallel Tracking and Mapping for Small AR Workspaces

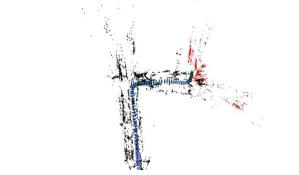
ISMAR 2007 video results

Georg Klein and David Murray Active Vision Laboratory University of Oxford

ORB-SLAM

- Supports both monocular and stereo cameras
- Feature based
 - FAST corners + ORB descriptors
 - ORB: binary descriptor, very fast to compute and match (Hamming distance)
 - Jointly optimizes poses & structure (sliding window BA)
- Same workflow as PTAM (keyframe based, alternation of localization and mapping as independent threads)
- Includes:
 - Loop closing
 - Relocalization
 - Final optimization
- **Real-time (30Hz)**, however global optimization is not done in real time but asynchronously every once in a while



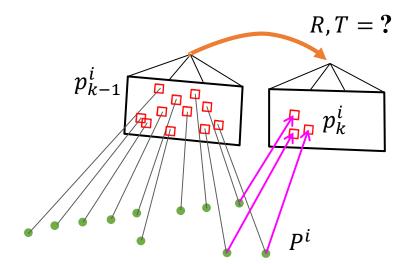


Indirect vs Direct Methods

Indirect methods

- 1. Extract & match features + 3-point RANSAC
- 2. Bundle Adjust by minimizing the **Reprojection Error**:

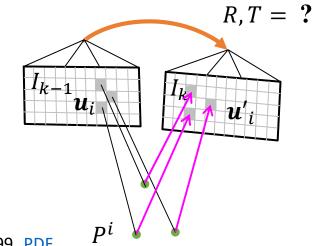
$$P^{i}, R, T = \arg\min_{P^{i}, R, T} \sum_{i=1}^{N} \rho\left(p_{k}^{i} - \pi\left(P^{i}, K, R, T\right)\right)$$



• Direct methods

1. No feature extraction & no RANSAC needed. Instead, directly minimize **Photometric Error**:

$$P^{i}, R, T = \arg \min_{P^{i}, R, T} \sum_{i=1}^{N} \rho \left(I_{k-1} (p_{k-1}^{i}) - I_{k} \left(\pi (P^{i}, K, R, T) \right) \right)$$



What are their pros and cons?

Irani, Anandau, All about direct methods, Springer'99. PDF

Indirect vs Direct Methods

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- Can cope with large frame-to-frame motions (large basin of convergence)
- Slow due to costly feature extraction, matching, and outlier removal (e.g., RANSAC)

- ✓ All information in the image can be exploited (higher accuracy, higher robustness to motion blur and weak texture (i.e., weak gradients))
- Increasing the camera frame-rate reduces computational cost per frame (no RANSAC needed)
- ✓ Very sensitive to intial value → limited frame-to-frame motion (small basin of convergence)

Direct Methods: Dense, Semi-dense, Sparse

Dense methods track every pixel Semi-Dense methods track only edges Sparse methods track sparse pixels



In a VGA image: 300'000+ pixels

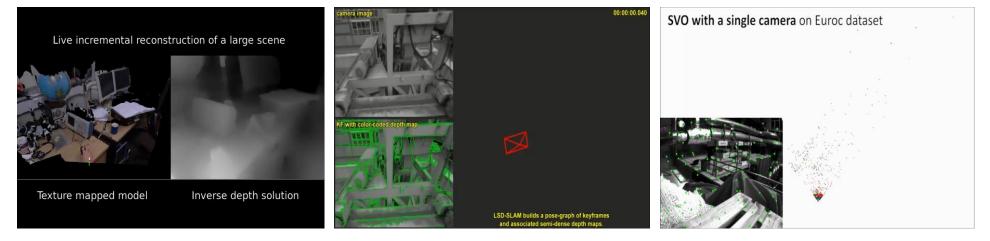
In a VGA image: ~10,000 pixels

In a VGA image: ~2,000 pixels

Forster, Zhang, Gassner, Werlberger, Scaramuzza, SVO: Semi Direct Visual Odometry for Monocular and Multi-Camera Systems, IEEE Transactions on Robotics (T-RO), 2017. <u>PDF</u>.]

Direct Methods: Dense, Semi-dense, Sparse

Dense methods track every pixel Semi-Dense methods track only edges Sparse methods track sparse pixels



In a VGA image: 300'000+ pixels

In a VGA image: ~10,000 pixels

In a VGA image: ~2,000 pixels e.g., 120 feature patches \times (4 \times 4 pixels per patch)

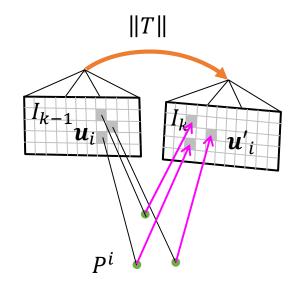
DTAM [Newcombe '11], REMODE [Pizzoli'14]

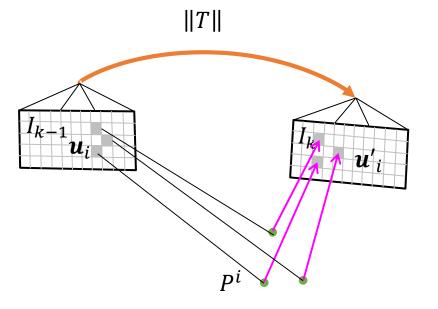
LSD-SLAM [Engel'14]

SVO [Forster'14], DSO [Engel'17]

Direct Methods: Dense, Semi-dense, Sparse

• What is the influence of the motion baseline on the convergence rate of direct methods?



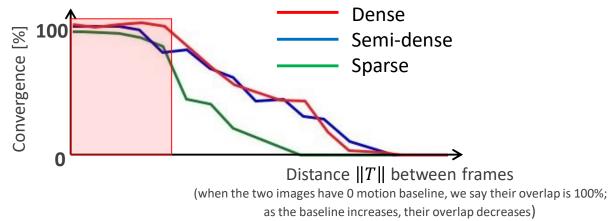


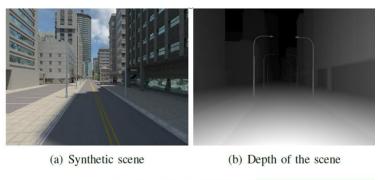
For small motion baselines, ||T||, the photometric error is usually small For large motion baselines, ||T||, the photometric error is usually large

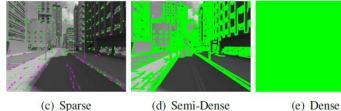
Forster, Zhang, Gassner, Werlberger, Scaramuzza, SVO: Semi Direct Visual Odometry for Monocular and Multi-Camera Systems, IEEE Transactions on Robotics (T-RO), 2017. <u>PDF</u>.]

What is the influence of the motion baseline on the convergence rate of direct methods?

We can use **photorealistic simulation** to answer this question by generating thousands of data







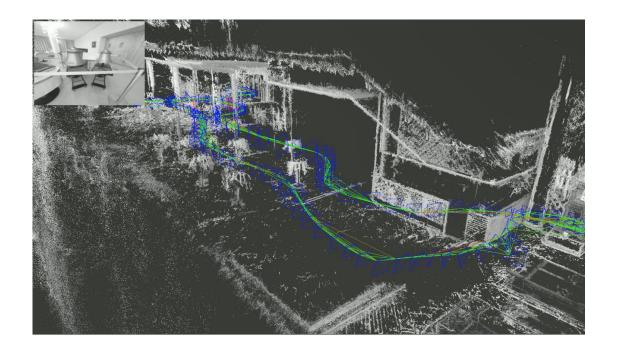
Simulated dataset from here

- Findings:
 - Dense and Semi-dense behave similarly
 - Weak gradients are not informative for the optimization
 - Dense methods are only useful with motion blur, defocus, and weak- texture regions
 - Sparse methods behave equally well as dense or semi-dense methods for small motion baselines

Forster, Zhang, Gassner, Werlberger, Scaramuzza, SVO: Semi Direct Visual Odometry for Monocular and Multi-Camera Systems, IEEE Transactions on Robotics (T-RO), 2017. <u>PDF</u>.]

LSD-SLAM

- Supports both **monocular and stereo** cameras
- **Direct** (photometric error) + **Semi-Dense** formulation
 - 3D structure represented as semi-dense depth map
 - Minimizes photometric error
 - Separateley optimizes poses & structure (sliding window)
- Same workflow as PTAM (keyframe based, alternation of localization and mapping as independent threads)
- Includes:
 - Loop closing
 - Relocalization
 - Final optimization
- **Real-time (30Hz)**, however global optimization is not done in real time but asynchronously every once in a while

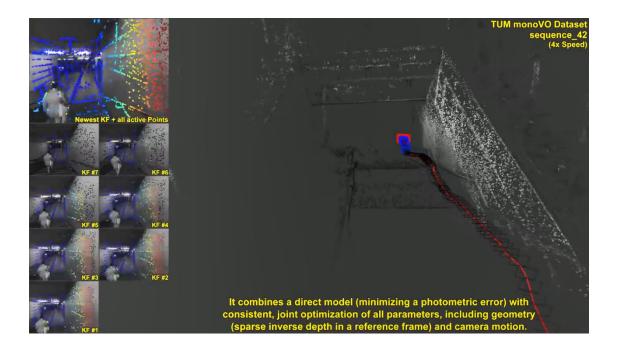


DSO

- Supports both **monocular and stereo** cameras
- **Direct** (photometric error) + **Sparse** formulation
 - 3D structure represented as sparse large gradients' depth map
 - Minimizes photometric error
 - Jointly optimizes poses & structure (sliding window)
 - Incorporates photometric correction to compensate exposure time change ($\Delta t_{k-1}, \Delta t_k$)

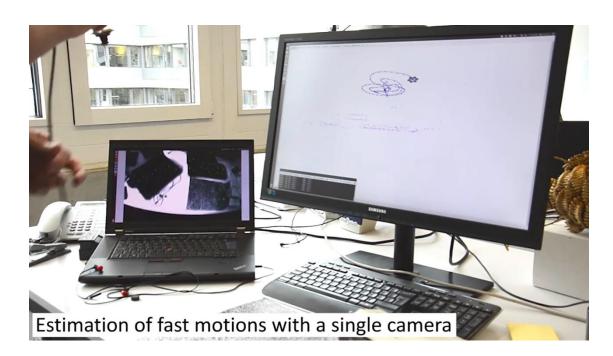
$$P^{i}, R, K = \arg\min_{P^{i}, R, K} \sum_{i=1}^{N} \rho \left(I_{k-1}(p_{k-1}^{i}) - \frac{\Delta t_{k-1}}{\Delta t_{k}} I_{k}(\pi(P^{i}, K, R, T)) \right)$$

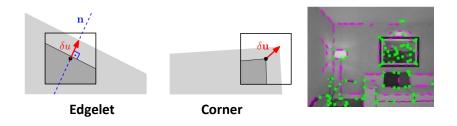
- Same workflow as PTAM (keyframe based, alternation of localization and mapping as independent threads)
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SVO

- Supports both **monocular, stereo, multi-camera systems** as well as omnidirectional models (fisheye and catadioptric)
- Combines indirect + direct methods
 - Direct methods for frame-to-frame motion estimation
 - Indirect methods for frame-to-keyframe pose refinement
- Mapping
 - Probabilistic depth estimation (heavy-tail Gaussian distribution)
- Includes:
 - Loop closing,
 - Relocalization,
 - Final optimization
- Same workflow as PTAM (keyframe based, alternation of localization and mapping as independent threads)
- Faster than real-time: up to 400 fps on i7 laptops and 100 fps on smartphone PCs (Odroid (ARM)) or NVIDIA Jetson



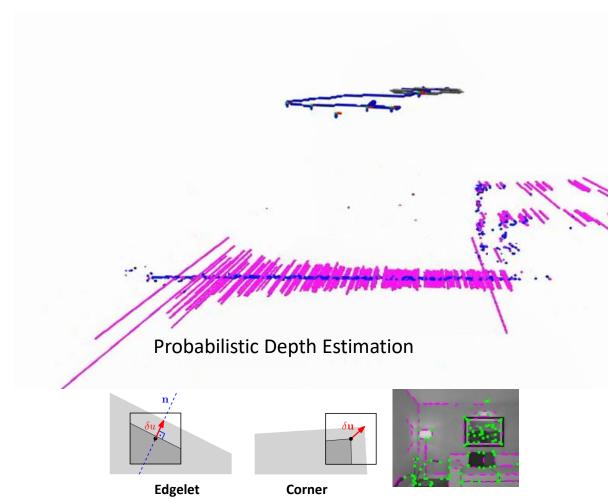




Forster, Zhang, Gassner, Werlberger, Scaramuzza, SVO: Semi Direct Visual Odometry for Monocular and Multi-Camera Systems, IEEE Transactions on Robotics (T-RO), 2017. PDF, code, videos.

SVO

- Supports both **monocular, stereo, multi-camera systems** as well as omnidirectional models (fisheye and catadioptric)
- Combines indirect + direct methods
 - Direct methods for frame-to-frame motion estimation
 - Indirect methods for frame-to-keyframe pose refinement
- Mapping
 - Probabilistic depth estimation (heavy-tail Gaussian distribution)
- Includes:
 - Loop closing,
 - Relocalization,
 - Final optimization
- Same workflow as PTAM (keyframe based, alternation of localization and mapping as independent threads)
- Faster than real-time: up to 400 fps on i7 laptops and 100 fps on smartphone PCs (Odroid (ARM)) or NVIDIA Jetson





Forster, Zhang, Gassner, Werlberger, Scaramuzza, SVO: Semi Direct Visual Odometry for Monocular and Multi-Camera Systems, IEEE Transactions on Robotics (T-RO), 2017. <u>PDF, code, videos</u>.

Processing times of ORB-SLAM, LSD-SLAM, DSO, SVO

	Mean	CPU@20 fps
SVO Mono	2.53	55 ±10%
ORB Mono SLAM (No loop closure LSD Mono SLAM (No loop closure DSO		187 ±32% 236 ±37% 181 ±27%
	1	1
	Processing tin in millisecon	·

Forster, Zhang, Gassner, Werlberger, Scaramuzza, SVO: Semi Direct Visual Odometry for Monocular and Multi-Camera Systems, IEEE Transactions on Robotics (T-RO), 2017. <u>PDF, code, videos</u>.

SVO and its derivatives are used today in many of products...

- DJI drones
- Magic Leap AR headsets
- Oculus VR headsets
- Huawei phones
- Nikon cameras







Autonomous quadrotor navigation in dynamic scenes (down-looking camera) (running on Odroid U3 board (ARM Cortex A9 at 90fps)



Throw-and-go (2015) (inspired many products, like <u>DJI Tello drone</u>)



20 m/s obstacle free autonomous quadrotor flight at DARPA FLA (2015)

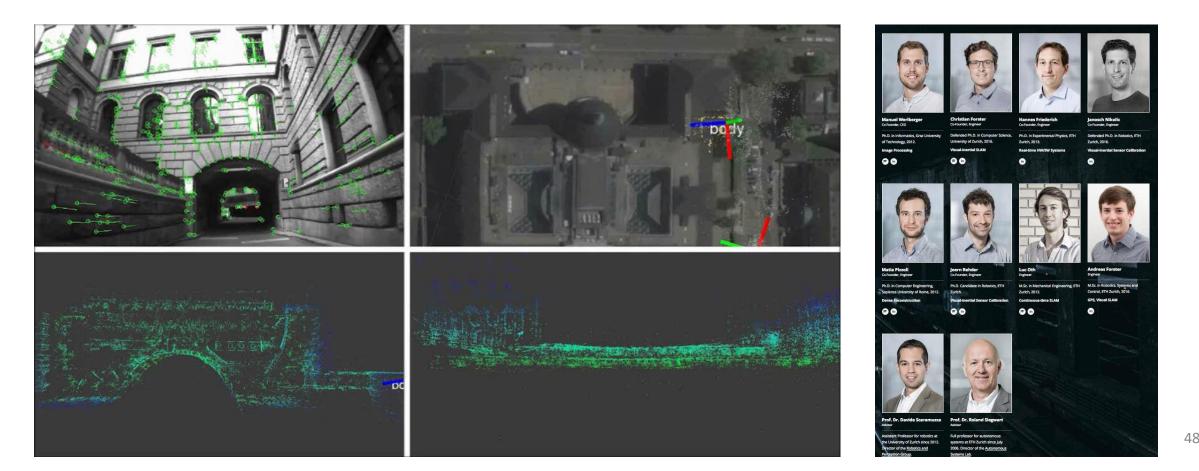


Virtual Reality with SVO running on an iPhone 6 (with company Dacuda at CES 2017. Dacuda is today Magic Leap Zurich)



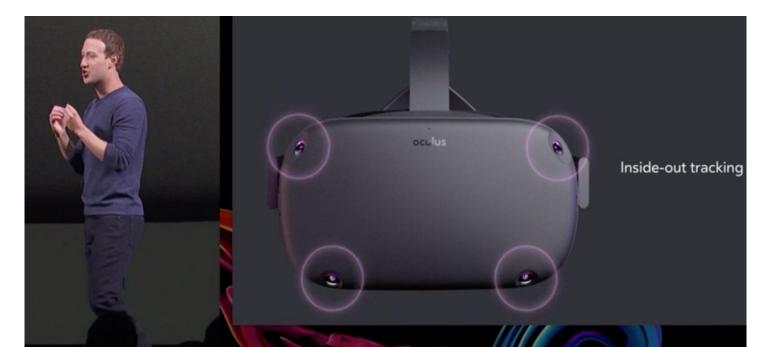
Startup: "Zurich-Eye" – Today: Facebook-Oculus Zurich

- Vision-based Localization and Mapping systems for mobile robots
- Born in Sep. 2015, became Facebook-Oculus Zurich in Sep. 2016. Today, 200 employees.



Startup: "Zurich-Eye" – Today: Facebook-Oculus Zurich

- Vision-based Localization and Mapping systems for mobile robots
- Born in Sep. 2015, became Facebook-Oculus Zurich in Sep. 2016. Today, 200 employees.
- In 2018, Zurich-Eye launched **Oculus Quest** (2 million units sold so far)
- Christian Forster (Facebook Zurich & co-founder of Zurich-Eye) gave a lecture on Nov. 26, 2020, which will be shared on OLAT.



Things to remember

- Hierarchical SFM
- VO flowchart
 - Monocular VO
 - Stereo VO
 - Keyframe selection
- Bundle adjustment vs pose-graph optimization
- Indirect vs direct methods
- Direct methods: Dense, semi-dense, and sparse formulations
- Popular open-source VO algorithms

Readings

- Scaramuzza, D., Fraundorfer, F., Visual Odometry: Part I The First 30 Years and Fundamentals, *IEEE Robotics and Automation Magazine*, Volume 18, issue 4, 2011. <u>PDF</u>
- Fraundorfer, F., Scaramuzza, D., Visual Odometry: Part II Matching, Robustness, and Applications, *IEEE Robotics and Automation Magazine*, Volume 19, issue 1, 2012. <u>PDF</u>
- C. Cadena, L. Carlone, H. Carrillo, Y. Latif, D. Scaramuzza, J. Neira, I.D. Reid, J.J. Leonard, Past, Present, and Future of Simultaneous Localization and Mapping: Toward the Robust-Perception Age, IEEE Transactions on Robotics, Vol. 32, Issue 6, 2016. <u>PDF</u>

Understanding Check

Are you able to answer the following questions:

- Bundle Adjustment and Pose Graph Optimization. Mathematical expressions and illustrations. Pros and cons.
- Are you able to describe hierarchical and sequential SFM for monocular VO?
- What are the building blocks of visual odometry and SLAM?
- What are keyframes? Why do we need them and how can we select them?
- Are you able to define loop closure detection? Why do we need loops? How can we detect loop closures? (make link to other lectures)
- Are you able to describe the differences between feature-based methods and direct methods?
- Sparse vs semi-dense vs dense. What are their pros and cons?
- Are you able to provide a list of the most popular open source VO and VSLAM algorithms?
- Difference between SFM, VO, SLAM (see also lecture 01)