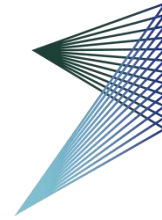




University of
Zurich^{UZH}

ETH zürich

Institute of Informatics – Institute of Neuroinformatics



ROBOTICS &
PERCEPTION
GROUP

Lecture 13

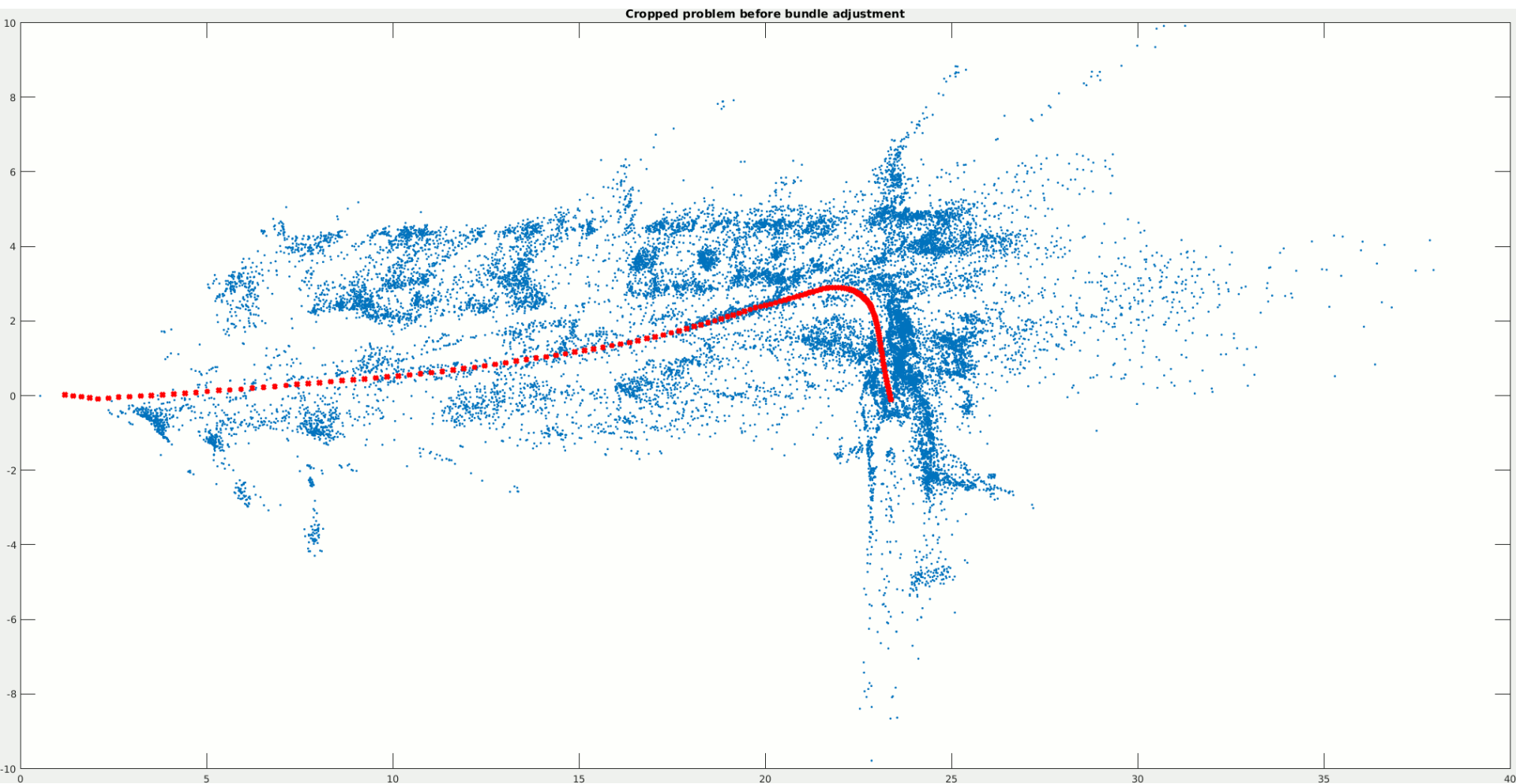
Visual Inertial Fusion

Davide Scaramuzza

<http://rpg.ifi.uzh.ch/>

Lab Exercise 9 – Today afternoon

- Room ETH HG E 1.1 from 13:15 to 15:00
- Work description: Bundle Adjustment



Visual Inertial Odometry

References:

- Scaramuzza, Zhang, **Visual-Inertial Odometry of Aerial Robots**, Encyclopedia of Robotics, Springer, 2019, [PDF](#)
- Huang, **Visual-inertial navigation: A concise review**, International conference on Robotics and Automation, 2019. [PDF](#).

Outline

➤ Introduction

➤ IMU model and Camera-IMU system

➤ Visual Inertial (VIO) Fusion

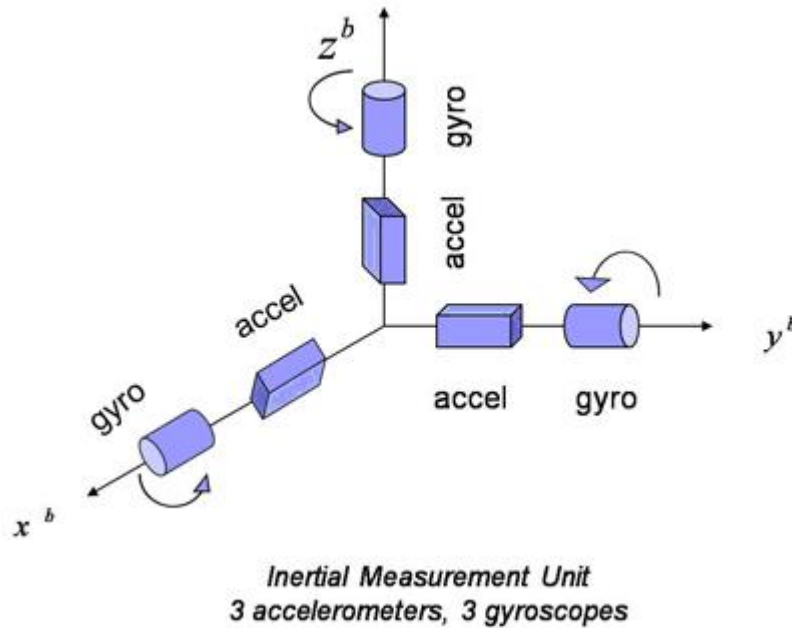
- Closed-form solution
- Filtering approaches
- Smoothing methods
 - Fixed-lag Smoothing (aka sliding window estimators)
 - Full smoothing methods

➤ Camera-IMU extrinsic calibration and Synchronization

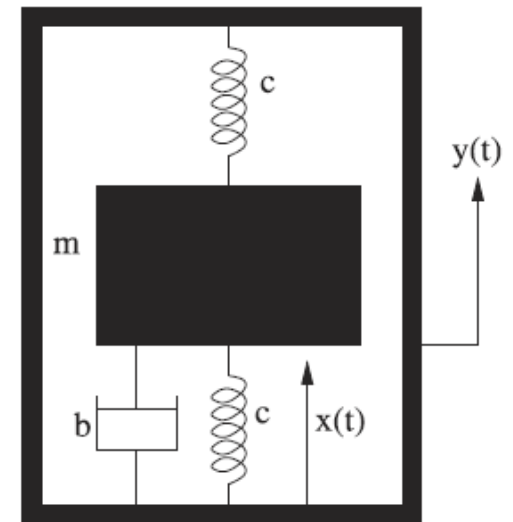
What is an IMU?

➤ Inertial Measurement Unit

- Gyroscope: Angular velocity
- Accelerometer: Linear Accelerations



Mechanical Gyroscope



Mechanical Accelerometer

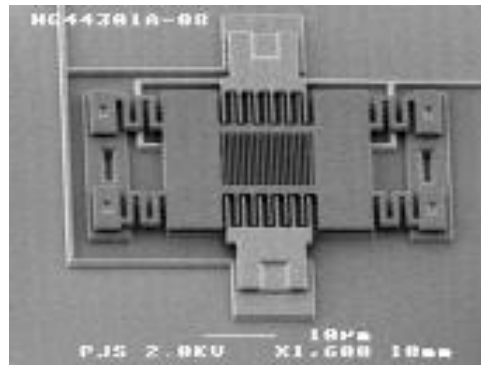
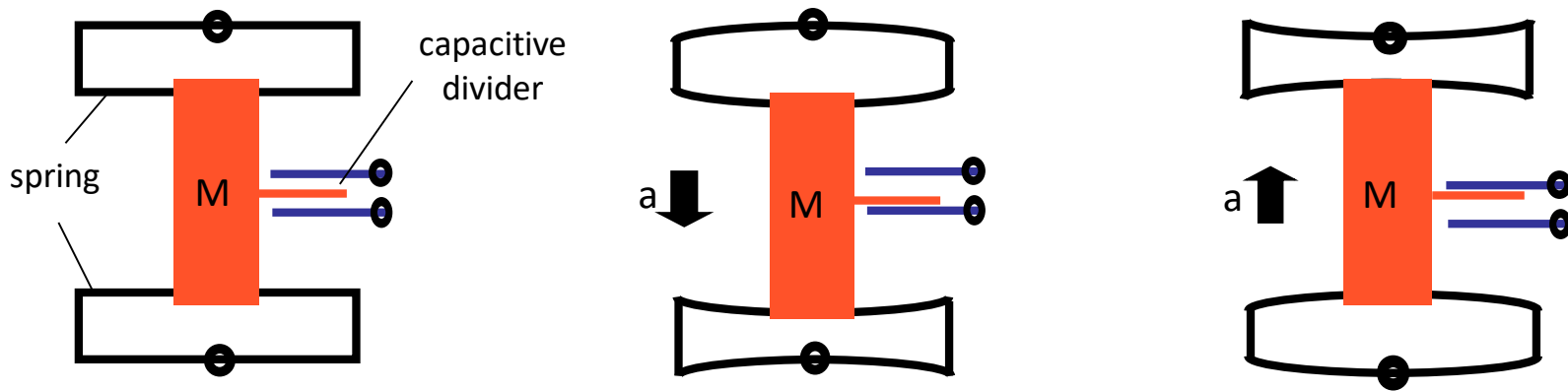
What is an IMU?

- Different categories
 - Mechanical (\$100,000-1M)
 - Optical (\$20,000-100k)
 - MEMS (from 1\$ (phones) to 1,000\$ (higher cost because they have a microchip running a Kalman filter))
- For small mobile robots & drones: MEMS IMU are mostly used
 - Cheap
 - Power efficient
 - Light weight and solid state



MEMS Accelerometer

A spring-like structure connects the device to a seismic mass vibrating in a capacitive divider. A capacitive divider converts the displacement of the seismic mass into an electric signal. Damping is created by the gas sealed in the device.



MEMS Gyroscopes

- MEMS gyroscopes measure the Coriolis forces acting on MEMS vibrating structures (tuning forks, vibrating wheels, or resonant solids)
- Their working principle is similar to the haltere of a fly
- Haltere are small structures of some two-winged insects, such as flies. They are flapped rapidly and function as gyroscopes, informing the insect about rotation of the body during flight.



Why IMU?

- Monocular vision is scale ambiguous.
- Pure vision is not robust enough
 - Low texture
 - High dynamic range
 - High speed motion

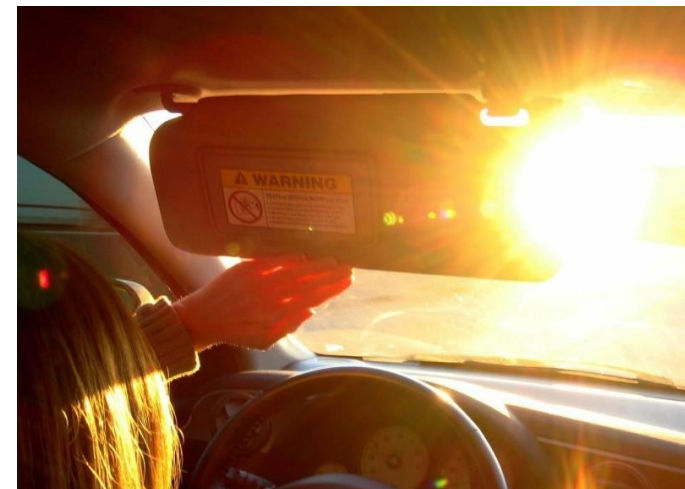
Robustness is a critical issue: Tesla accident

“The autopilot sensors on the Model S failed to distinguish a white tractor-trailer crossing the highway against a bright sky.” [[The Guardian](#)]

Motion blur



Dynamic Range



Why vision?

➤ Pure IMU integration will lead to large drift (especially cheap IMUs)

- Will see later mathematically
- Intuition
 - Integration of angular velocity to get orientation: if there is a bias in angular velocity, the error is **proportional to t**
 - Double integration of acceleration to get position: if there is a bias in acceleration, the error of position is **proportional to t²**
 - Worse, the actual position error also depends on the orientation error (see later).

	Accelerometer Bias Error		Horizontal Position Error [m]			
Grade	[mg]		1s	10s	60s	1hr
Navigation	0.025		0.13 mm	12 mm	0.44 m	1.6 km
Tactical	0.3		1.5 mm	150 mm	5.3 m	19 km
Industrial	3		15 mm	1.5 m	53 m	190 km
Automotive	125		620 mm	60 m	2.2 km	7900 km

Automotive,
Smartphone,
& Drone
accelerometers

Why visual inertial fusion?

IMU and vision are complementary

Cameras

- ✓ Precise in slow motion
- ✓ Rich information for other tasks
- ✗ Limited output rate (~100 Hz)
- ✗ Scale ambiguity in monocular setup
- ✗ Lack of robustness to HDR and high speed

IMU

- ✓ Robust
- ✓ High output rate (~1,000 Hz)
- ✓ Accurate at high acceleration
- ✓ Can predict next feature position
- ✗ Large relative uncertainty when at low acceleration/angular velocity
- ✗ Ambiguity in gravity / acceleration

What cameras and IMU have in common: both estimate the pose incrementally (known as dead-reckoning), which suffers from drifting over time. **Solution: loop detection and loop closure**

Outline

➤ Introduction

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- Closed-form solution
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 - Full smoothing methods

➤ Camera-IMU extrinsic calibration and Synchronization

IMU model: Measurement Model

- Measures angular velocity and acceleration in the body frame:

$$\begin{aligned}
 \boxed{\mathbf{{}_B \tilde{\boldsymbol{\omega}}_{WB}(t)}} &= \mathbf{{}_B \boldsymbol{\omega}_{WB}(t)} + \boxed{\mathbf{b}^g(t) + \mathbf{n}^g(t)} \\
 \boxed{\mathbf{{}_B \tilde{\mathbf{a}}_{WB}(t)}} &= \mathbf{R}_{BW}(t) \left(\mathbf{{}_W \mathbf{a}_{WB}(t)} - \mathbf{{}_W \mathbf{g}} \right) + \boxed{\mathbf{b}^a(t) + \mathbf{n}^a(t)}
 \end{aligned}$$

measurements true $\boldsymbol{\omega}$ and \mathbf{a} to estimate IMU biases + noise

where the superscript g stands for Gyroscope and a for Accelerometer

Notations:

- Left subscript: reference frame in which the quantity is expressed
- Right subscript $\{Q\}\{Frame1\}\{Frame2\}$: Q of Frame2 with respect to Frame1
- Biases and noise are expressed in the body frame

IMU model: Noise Property

➤ Additive Gaussian white noise: $\mathbf{n}^g(t)$, $\mathbf{n}^a(t)$

➤ Bias: $\mathbf{b}^g(t)$, $\mathbf{b}^a(t)$

$$\dot{\mathbf{b}}(t) = \sigma_b \mathbf{w}(t) \quad \mathbf{w}(t) \sim \mathbf{N}(0, 1)$$

i.e., the derivative of the bias is white Gaussian noise

Some facts about IMU biases:

- They can change due to temperature change and mechanical pressure
- They can change every time the IMU is started
- Good news: they can be estimated

Trawny, Nikolas, and Stergios I. Roumeliotis. "Indirect Kalman filter for 3D attitude estimation."

<https://github.com/ethz-asl/kalibr/wiki/IMU-Noise-Model>

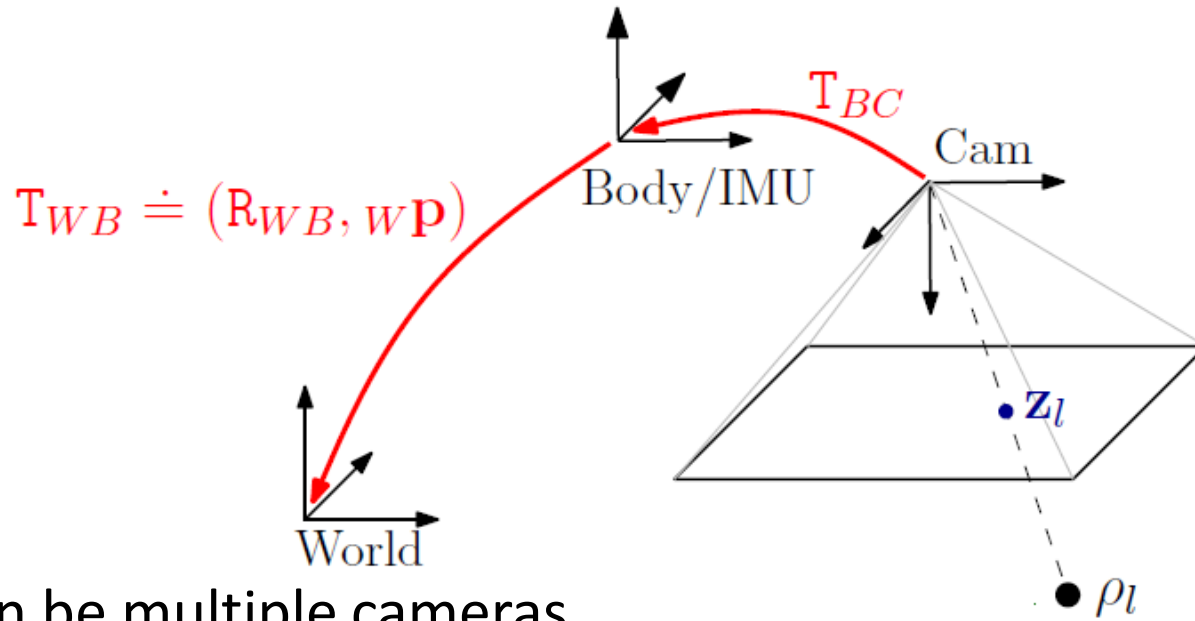
IMU model: Integration

$$\mathbf{p}_{\mathbf{w}t_2} = \mathbf{p}_{\mathbf{w}t_1} + (t_2 - t_1) \mathbf{v}_{\mathbf{w}t_1} + \int_{t_1}^{t_2} \int_{t_1}^t \left(\mathbf{R}_{\mathbf{w}t}(t) (\tilde{\mathbf{a}}(t) - \mathbf{b}^a(t)) + {}_{\mathbf{w}}\mathbf{g} \right) dt^2$$

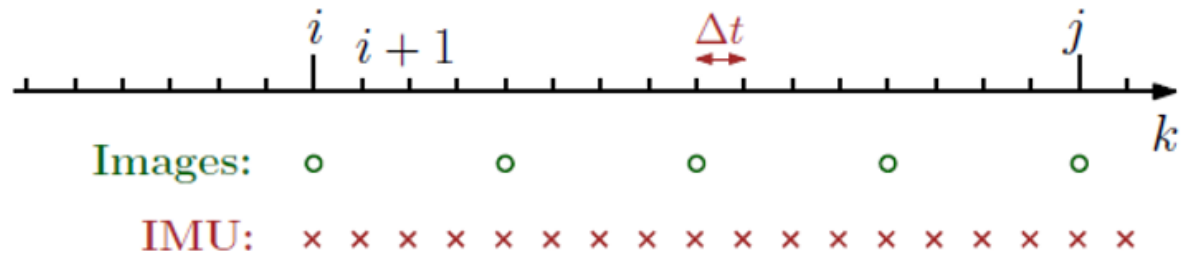
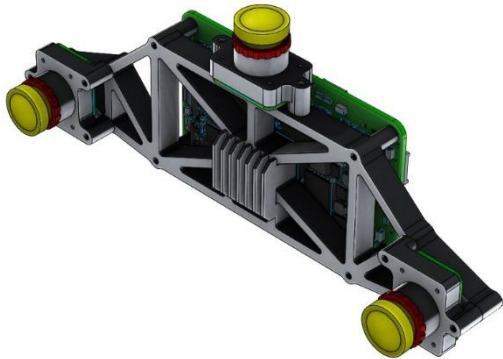
per component: $\{t\}$ stands for $\{B\}$ ody frame at time t

- Depends on initial position and velocity
- The rotation $R(t)$ is computed from the gyroscope

Camera-IMU System



There can be multiple cameras.



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Visual Inertial (VIO) Fusion

Different paradigms

➤ **Loosely coupled:**

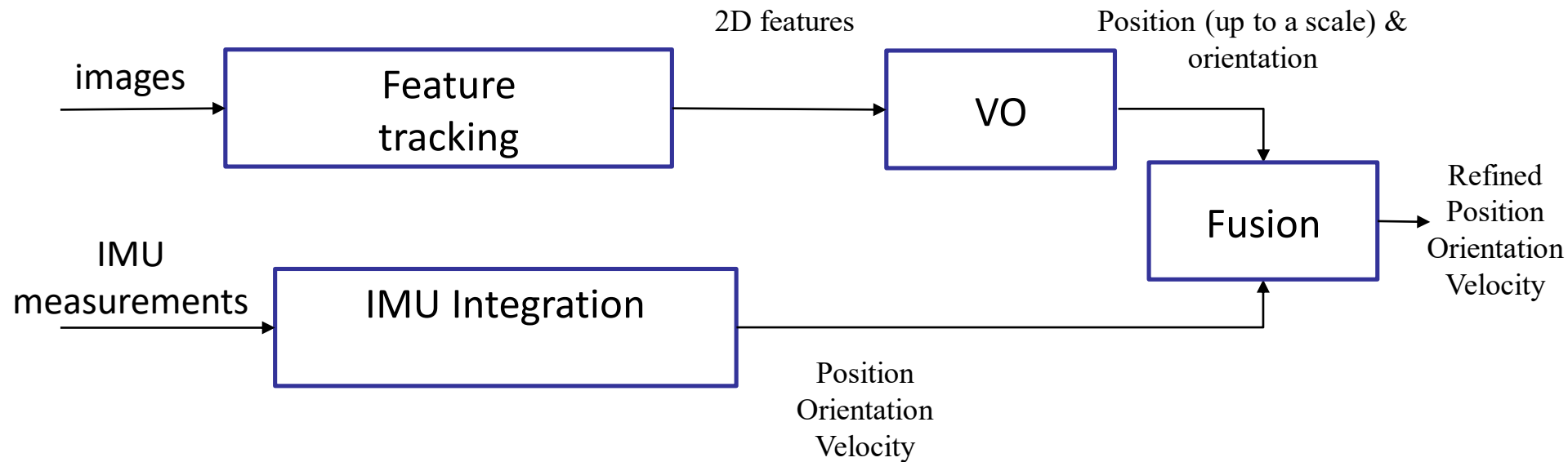
- Treats VO and IMU as two separate (not coupled) black boxes
 - Each black box estimates pose and velocity from visual (up to a scale) and inertial data (absolute scale)

➤ **Tightly coupled:**

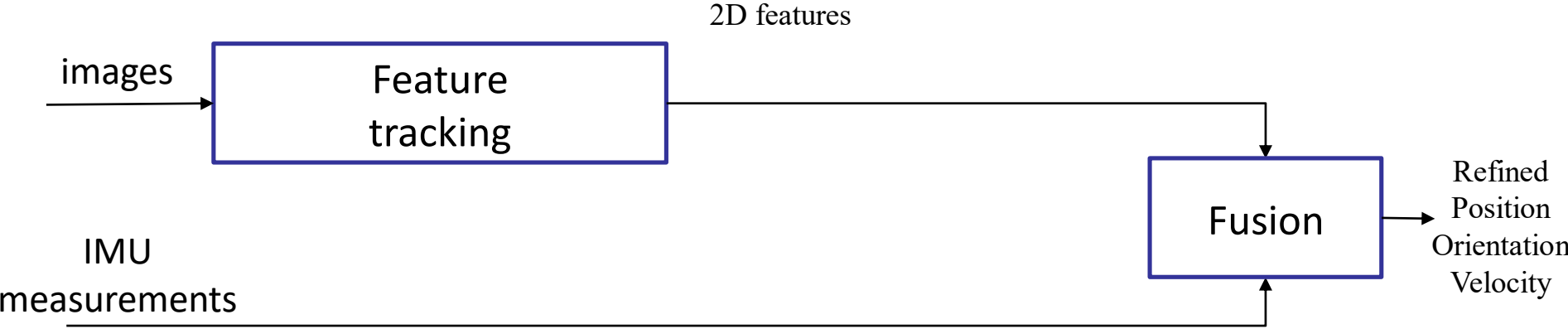
- Makes use of the raw sensors' measurements:
 - 2D features
 - IMU readings
 - **More accurate**
 - **More implementation effort**

In the following slides, we will only see **tightly coupled approaches**

The Loosely Coupled Approach



The Tightly Coupled Approach



Filtering: Visual Inertial Formulation

System states:

Tightly coupled: $\mathbf{X} = \left[\mathbf{}^w\mathbf{p}(t); \mathbf{q}_{WB}(t); \mathbf{}^w\mathbf{v}(t); \mathbf{b}^a(t); \mathbf{b}^g(t); \mathbf{}^w\mathbf{L}_1; \mathbf{}^w\mathbf{L}_2; \dots; \mathbf{}^w\mathbf{L}_K \right]$

Loosely coupled: $\mathbf{X} = \left[\mathbf{}^w\mathbf{p}(t); \mathbf{q}_{WB}(t); \mathbf{}^w\mathbf{v}(t); \mathbf{b}^a(t); \mathbf{b}^g(t) \right]$

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Closed-form Solution (1D case)

- The absolute pose x is known up to a scale s , thus

$$x = s\tilde{x}$$

- From the IMU

$$x = x_0 + v_0(t_1 - t_0) + \iint_{t_0}^{t_1} a(t)dt$$

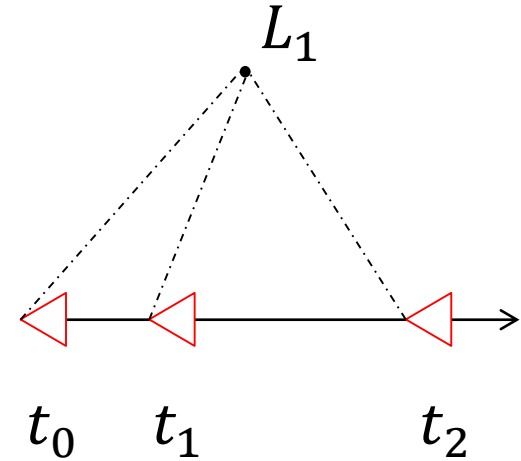
- By equating them

$$s\tilde{x} = x_0 + v_0(t_1 - t_0) + \iint_{t_0}^{t_1} a(t)dt$$

- As shown in [Martinelli'14], for 6DOF, both s and v_0 can be determined in closed form from a **single feature observation and 3 views**. x_0 can be set to 0.

Closed-form Solution (1D case)

$$\left\{ \begin{array}{l} s\widetilde{x}_1 = v_0(t_1 - t_0) + \iint_{t_0}^{t_1} a(t)dt \\ s\widetilde{x}_2 = v_0(t_2 - t_0) + \iint_{t_0}^{t_2} a(t)dt \end{array} \right.$$



$$\begin{bmatrix} \widetilde{x}_1 & (t_0 - t_1) \\ \widetilde{x}_2 & (t_0 - t_2) \end{bmatrix} \begin{bmatrix} s \\ v_0 \end{bmatrix} = \begin{bmatrix} \iint_{t_0}^{t_1} a(t)dt \\ \iint_{t_0}^{t_2} a(t)dt \end{bmatrix}$$

Closed-form Solution (general case)

- Considers N feature observations and 6DOF case
- Can be used to initialize filters and smoothers (which always need an initialization point)
- More complex to derive than the 1D case. But it also reaches a linear system of equations that can be solved using the pseudoinverse:

$$AX = S$$

X is the vector of unknowns:

- Absolute scale, s
- Initial velocity, v_0
- 3D Point distances (wrt the first camera)
- Direction of the gravity vector,
- Biases

A and S contain 2D feature coordinates, acceleration, and angular velocity measurements

$$A = \begin{bmatrix} T_2 & S_2 & \Gamma_2 & \mu_1^1 & 0_3 & 0_3 & -\mu_2^1 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_{33} & 0_{33} & 0_{33} & \mu_1^1 & -\mu_1^2 & 0_3 & -\mu_2^1 & \mu_2^2 & 0_3 & 0_3 & 0_3 & 0_3 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0_{33} & 0_{33} & 0_{33} & \mu_1^1 & 0_3 & -\mu_1^N & -\mu_2^1 & 0_3 & \mu_2^N & 0_3 & 0_3 & 0_3 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ T_{n_i} & S_{n_i} & \Gamma_{n_i} & \mu_1^1 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & -\mu_{n_i}^1 & 0_3 & 0_3 \\ 0_{33} & 0_{33} & 0_{33} & \mu_1^1 & -\mu_1^2 & 0_3 & 0_3 & 0_3 & 0_3 & -\mu_{n_i}^1 & \mu_{n_i}^2 & 0_3 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0_{33} & 0_{33} & 0_{33} & \mu_1^1 & 0_3 & -\mu_1^N & 0_3 & 0_3 & 0_3 & -\mu_{n_i}^1 & 0_3 & \mu_{n_i}^N \end{bmatrix}$$

- Martinelli, Vision and IMU data fusion: Closed-form solutions for attitude, speed, absolute scale, and bias determination, TRO'12
- Martinelli, Closed-form solution of visual-inertial structure from motion, Int. Journal of Comp. Vision, JCV'14
- Kaiser, Martinelli, Fontana, Scaramuzza, Simultaneous state initialization and gyroscope bias calibration in visual inertial aided navigation, IEEE RAL'17

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Different VIO fusion paradigms

Filtering	Fixed-lag Smoothing	Full smoothing
<p>Only updates the most recent state</p> <ul style="list-style-type: none"> (e.g., extended Kalman filter(EKF)) 	<p>Optimizes window of states</p> <ul style="list-style-type: none"> Marginalization Nonlinear least squares optimization 	<p>Optimizes all states</p> <ul style="list-style-type: none"> Nonlinear Least squares optimization
<p>×1 Linearization</p> <p>×Accumulation of linearization errors</p> <p>×Gaussian approximation of marginalized states</p> <p>✓Fastest</p>	<p>✓Re-Linearizes</p> <p>×Accumulation of linearization errors</p> <p>×Gaussian approximation of marginalized states</p> <p>✓Fast</p>	<p>✓Re-Linearizes</p> <p>✓Sparse Matrices</p> <p>✓Highest Accuracy</p> <p>×Slow (but fast with GTSAM)</p>

Filtering: Visual Inertial Formulation

System states:

Tightly coupled: $\mathbf{X} = \left[{}_w\mathbf{p}(t); \mathbf{q}_{wB}(t); {}_w\mathbf{v}(t); \mathbf{b}^a(t); \mathbf{b}^g(t); \boxed{{}_w\mathbf{L}_1; {}_w\mathbf{L}_2; \dots; {}_w\mathbf{L}_K} \right]$

Loosely coupled: $\mathbf{X} = \left[{}_w\mathbf{p}(t); \mathbf{q}_{wB}(t); {}_w\mathbf{v}(t); \mathbf{b}^a(t); \mathbf{b}^g(t) \right]$

Process Model: from IMU

- Integration of IMU states (rotation, position, velocity)
- Propagation of IMU noise
 - needed for calculating the Kalman filter gain

Filtering: ROVIO

- EKF state: $\mathbf{X} = \left[{}_W \mathbf{p}(t); \mathbf{q}_{WB}(t); {}_W \mathbf{v}(t); \mathbf{b}^a(t); \mathbf{b}^g(t); {}_W \mathbf{L}_1; {}_W \mathbf{L}_2; \dots; {}_W \mathbf{L}_K \right]$
- Minimizes the photometric error instead of the reprojection error

ROVIO: Robust Visual Inertial Odometry Using a Direct EKF-Based Approach

<http://github.com/ethz-asl/rovio>

Michael Bloesch, Sammy Omari, Marco Hutter, Roland Siegwart

Filtering: Problems

- Wrong **linearization point**:
 - Linearization depends on the current estimates of states, which may be erroneous

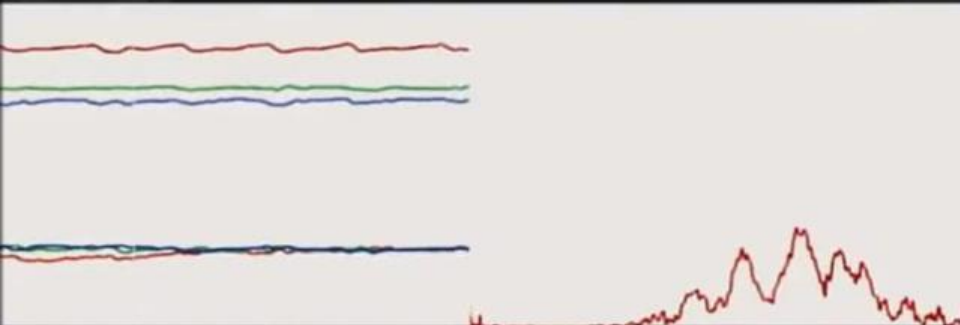
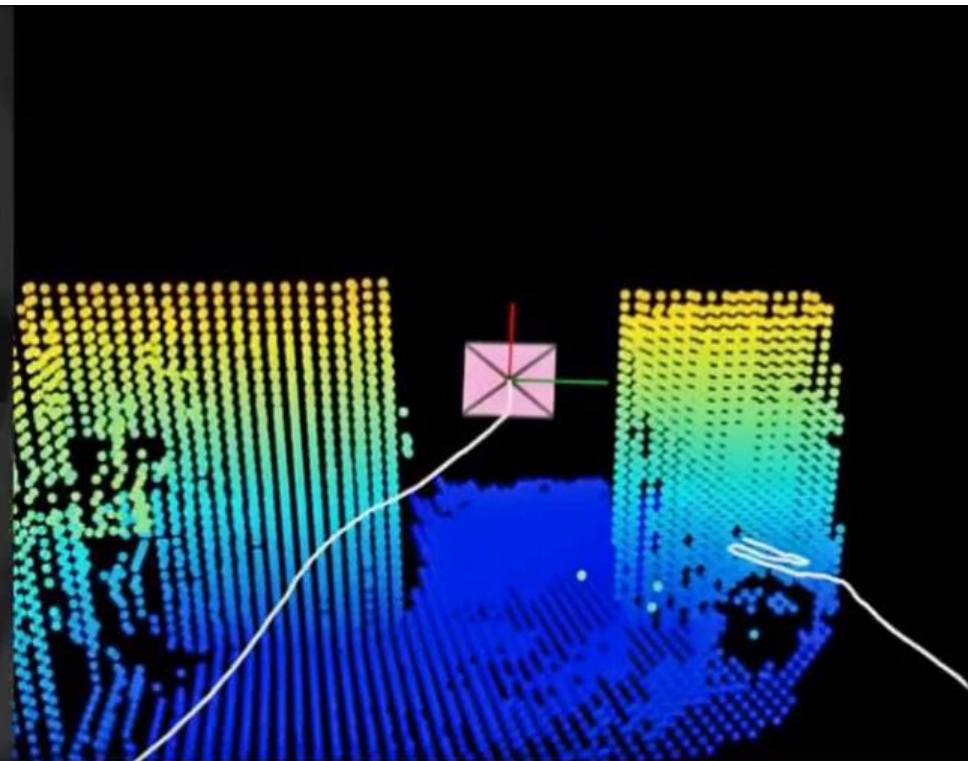
- **Complexity** of the EKF grows **quadratically** in the number of estimated landmarks,
 - a **few landmarks** (~20) are typically tracked to allow real-time operation

- **Alternative: MSCKF** [Mourikis & Roumeliotis, ICRA'07]: used in Google ARCore
 - Keeps a window of recent states and updates them using EKF
 - incorporate visual observations without including point positions into the states

Mourikis & Roumeliotis, A Multi-State Constraint Kalman Filter for Vision-aided Inertial Navigation, TRO'16

Li, Mingyang, and Anastasios I. Mourikis, High-precision, consistent EKF-based visual-inertial odometry, IJRR'13

Filtering: Google ARCore



Time (s): 18.98

FPS: 46.82

Updates: 23

Inliers: 111

Path length (m): 5.1

Position (m): 0.01, 1.09, -0.07

Dist. to origin (m): 1.1 (21.6 % of path)

Status: 3

Mourikis & Roumeliotis, A Multi-State Constraint Kalman Filter for Vision-aided Inertial Navigation, TRO'16

Li, Mingyang, and Anastasios I. Mourikis, High-precision, consistent EKF-based visual-inertial odometry, IJRR'13

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Smoothing methods

VIO solved as a **graph optimization** problem over:

$X = \{x_1, \dots, x_N\}$: Robot states a frame times (**position, velocity, orientation**)

$L = \{l_1, \dots, l_M\}$: **3D Landmarks**

$x_k = f(x_{k-1}, u)$ performs the integration of IMU measurements $u = (a, \omega)$

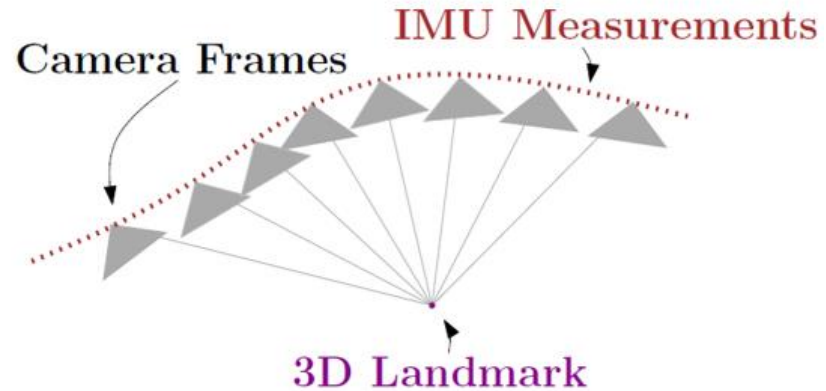
$z_{i_k} = \pi(x_k, l_i)$ performs the projection of the landmark l_i in the camera frame I_k

$\{X, L, b^a, b^g\} = \operatorname{argmin}_{\{X, L, b^a, b^g\}}$

$$\left\{ \underbrace{\sum_{k=1}^N \|f(x_{k-1}, u) - x_k\|_{\Lambda_k}^2}_{\text{IMU residuals}} + \underbrace{\sum_{k=1}^N \sum_{i=1}^M \|\pi(x_k, l_i) - z_{i_k}\|_{\Sigma_{i_k}}^2}_{\text{Reprojection residuals}} \right\}$$

Λ_k is the covariance from the IMU integration

Σ_{i_k} is the covariance from the noisy 2D feature measurements



Fixed-lag smoothing: OKVIS

OKVIS: Open Keyframe-based Visual-Inertial SLAM

A reference implementation of:

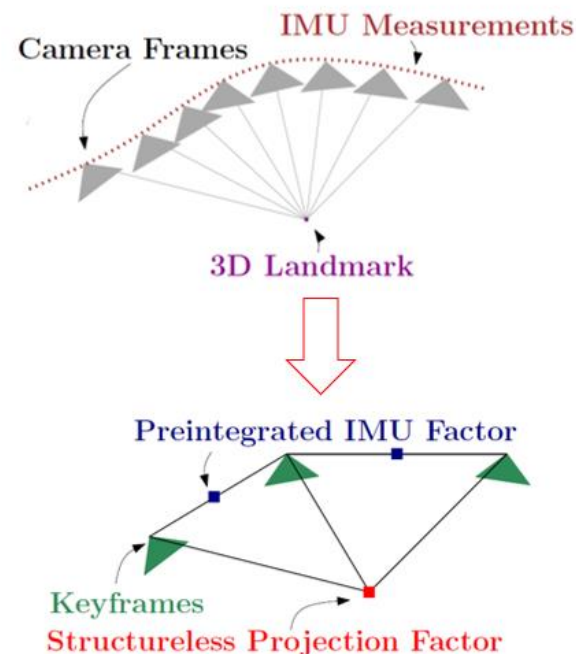
Stefan Leutenegger, Simon Lynen, Michael Bosse,
Roland Siegwart and Paul Timothy Furgale.
Keyframe-based visual-inertial odometry using
nonlinear optimization.

The International Journal of Robotics Research, 2015.

Full Smoothing: SVO+GTSAM & IMU Pre-integration

Solves the same optimization problem but:

- Keeps all the frames (from the start of the trajectory)
- To make the optimization efficient
 - it makes the graph sparser using keyframes
 - pre-integrates the IMU data between keyframes
- Optimization solved using factor graphs ([GTSAM](#))
 - Very fast because it only optimizes the poses that are affected by a new observation



$$\{X, L, b^a, b^g\} = \operatorname{argmin}_{\{X, L, b^a, b^g\}}$$

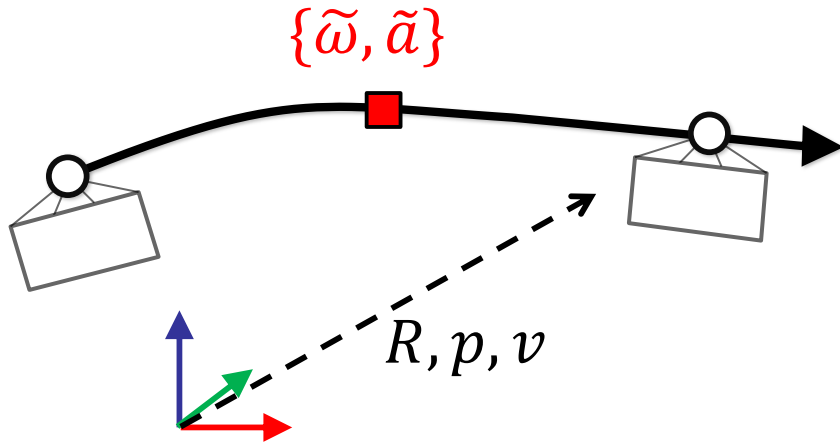
$$\left\{ \underbrace{\sum_{k=1}^N \|f(x_{k-1}, u) - x_k\|_{\Lambda_k}^2}_{\text{IMU residuals}} + \underbrace{\sum_{k=1}^N \sum_{i=1}^M \|\pi(x_k, l_i) - z_{i_k}\|_{\Sigma_{i_k}}^2}_{\text{Reprojection residuals}} \right\}$$

Problem with IMU integration

- The integration from k to $k+1$ is related to the state estimation at time k
- Idea: Preintegration

- Lupton, Sukkarieh. "Visual-inertial-aided navigation for high-dynamic motion in built environments without initial conditions."
- Forster, Carlone, Dellaert, Scaramuzza, "IMU preintegration on manifold for efficient visual-inertial maximum-a-posteriori estimation."

IMU Pre-Integration



Standard:

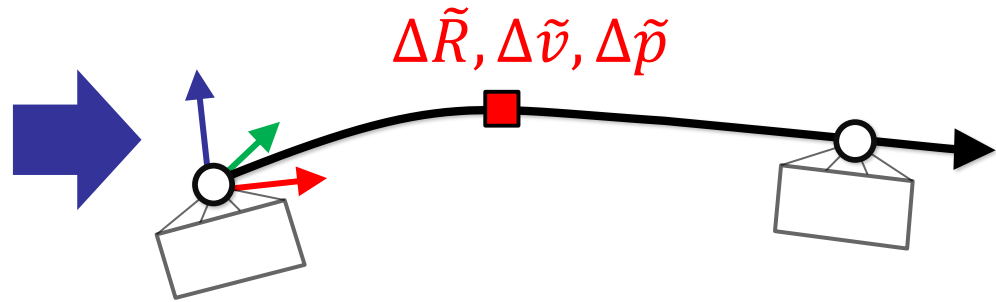
Evaluate **error in global frame**:

$$e_R = \hat{R}(\tilde{\omega}, R_{k-1})^T R_k$$

$$e_V = \hat{v}(\tilde{\omega}, \tilde{a}, v_{k-1}) - v_k$$

$$e_p = \underbrace{\hat{p}(\tilde{\omega}, \tilde{a}, p_{k-1})}_{\text{Predicted}} - \underbrace{p_k}_{\text{Estimate}}$$

Repeat integration when previous state changes!



Preintegration:

Evaluate **relative errors**:

$$e_R = \Delta \tilde{R}^T \Delta R$$

$$e_V = \Delta \tilde{v} - \Delta v$$

$$e_p = \Delta \tilde{p} - \Delta p$$

Preintegration of IMU deltas possible with **no initial condition required**. 37

Full Smoothing: SVO+GTSAM & IMU Pre-integration

IMU Preintegration on Manifold for Efficient Visual-Inertial Maximum-a-Posteriori Estimation

Christian Forster, Luca Carlone, Frank Dellaert, and Davide Scaramuzza



**University of
Zurich** ^{UZH}
Department of Informatics



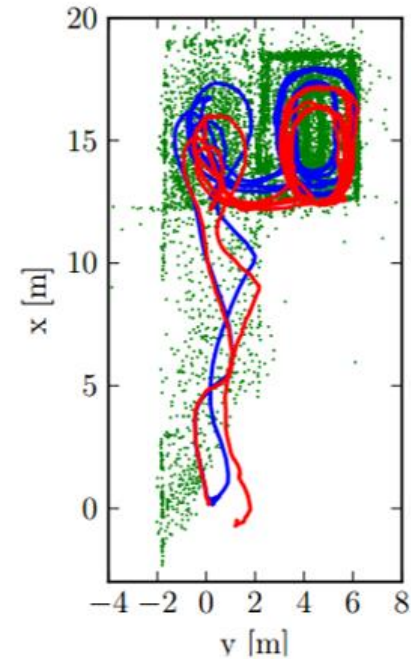
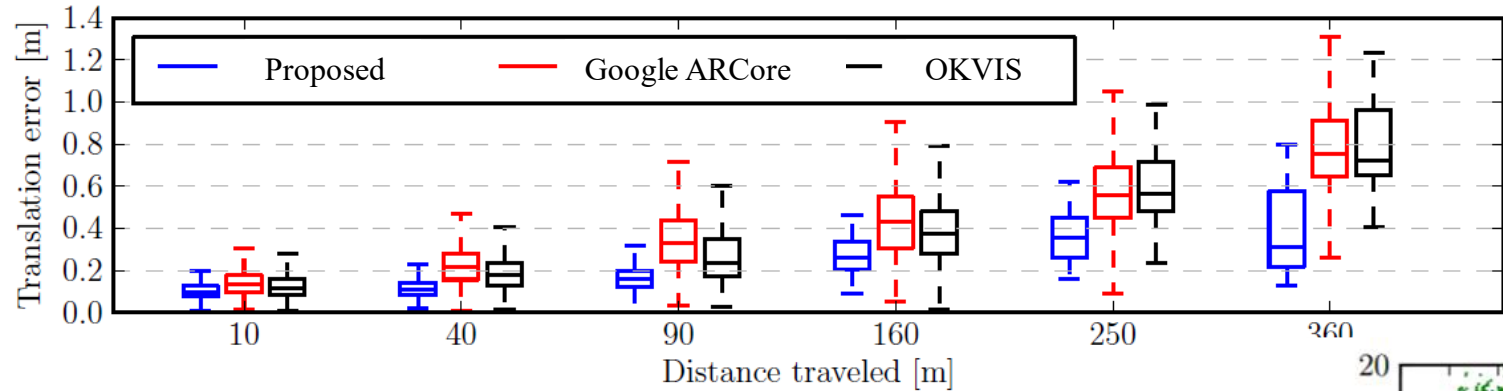
**Georgia Institute
of Technology**

rpg.ifi.uzh.ch

borg.cc.gatech.edu

SVO + IMU Preintegration

Accuracy: 0.1% of the travel distance



Recap

➤ Closed form solution:

- for 6DOF motion both s and v_0 can be determined **1 feature observation and at least 3 views** [Martinelli, TRO'12, IJCV'14, RAL'16]
- Can be used to **initialize filters and smoothers**

➤ Filters: *update only last state* → *fast if number of features is low (~20)*

- [Mourikis, ICRA'07, CVPR'08], [Jones, IJRR'11] [Kottas, ISER'12][Bloesch, IROS'15] [Wu et al., RSS'15], [Hesch, IJRR'14], [Weiss, JFR'13]
- **Open source: ROVIO** [Bloesch, IROS'15, IJRR'17], **MSCKF** [Mourikis, ICRA'07] (i.e., Google ARCore)

➤ Fixed-lag smoothers: *update a window of states* → *slower but more accurate*

- [Mourikis, CVPR'08] [Sibley, IJRR'10], [Dong, ICRA'11], [Leutenegger, RSS'13-IJRR'15]
- **Open source: OKVIS** [Leutenegger, RSS'13-IJRR'15], **VINS** [Qin, TRO'18]

➤ Full-smoothing methods: *update entire history of states* → *slower but more accurate*

- [Jung, CVPR'01] [Sterlow'04] [Bryson, ICRA'09] [Indelman, RAS'13] [Patron-Perez, IJCV'15] [Forster, RSS'15, TRO'16]
- **Open source: SVO+IMU** [Forster, TRO'17]

Open Problems: consistency

➤ Filters

- Linearization around different values of the same variable may lead to error

➤ Smoothing methods

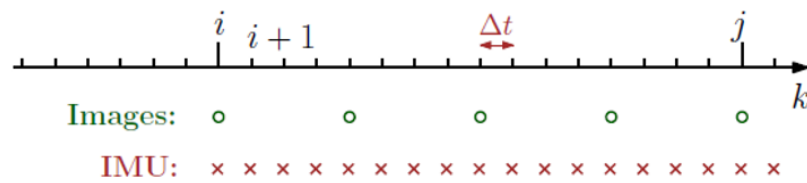
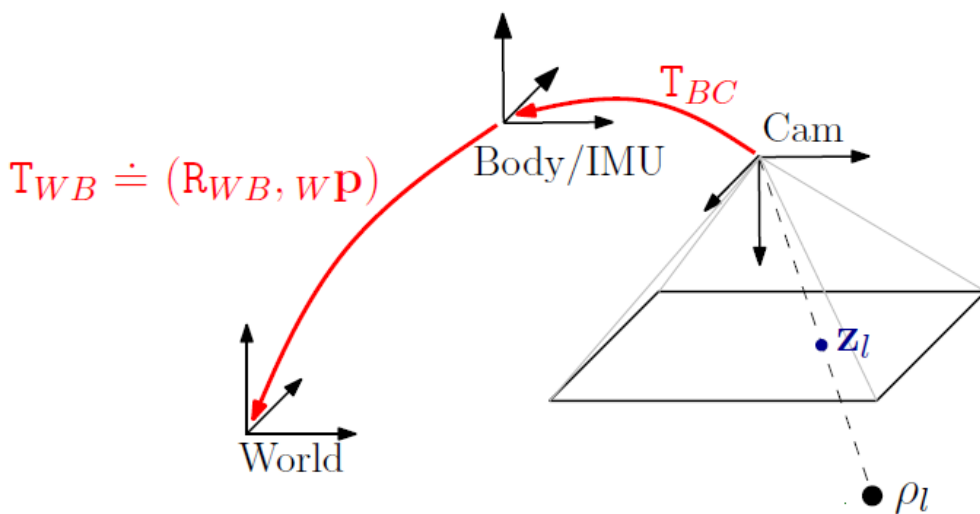
- May get stuck in local minima

Outline

- Introduction
- IMU model and Camera-IMU system
- Visual Inertial (VIO) Fusion
 - Closed-form solution
 - Filtering approaches
 - Smoothing methods
 - Fixed-lag Smoothing (aka sliding window estimators)
 - Full smoothing methods
- Camera-IMU extrinsic calibration and Synchronization

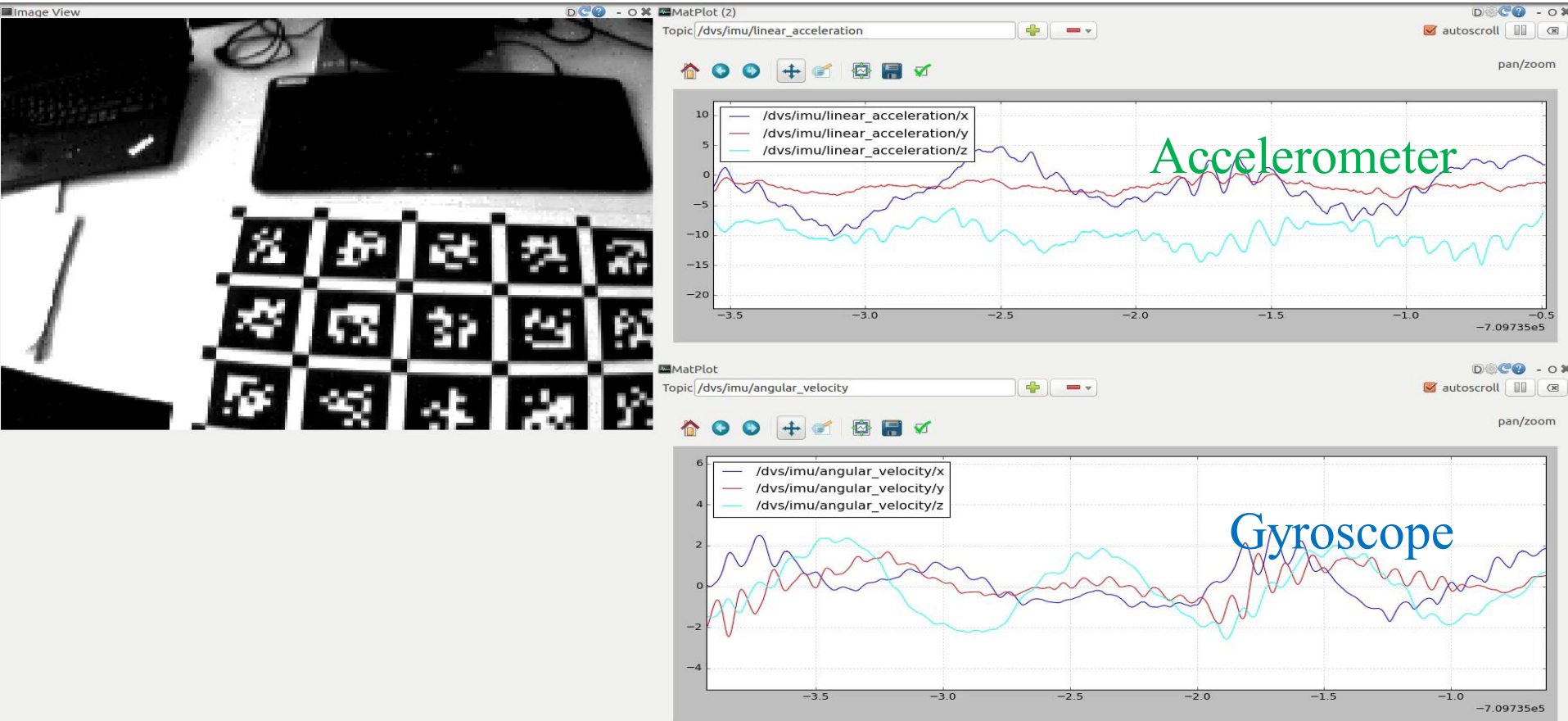
Camera-IMU calibration

- **Goal:** estimate the rigid-body transformation T_{BC} and delay t_d between a camera and an IMU rigidly attached. Assume that the camera has already been intrinsically calibrated.
- **Data:**
 - Image points of detected calibration pattern (checkerboard).
 - IMU measurements: accelerometer $\{a_k\}$ and gyroscope $\{\omega_k\}$.



Camera-IMU calibration - Example

- Data acquisition: Move the sensor in front of a static calibration pattern, exciting all degrees of freedom.



Furgale et al. "Unified Temporal and Spatial Calibration for Multi-Sensor Systems", IROS'13.

<https://github.com/ethz-asl/kalibr/wiki/camera-imu-calibration>

Camera-IMU calibration

➤ Approach: Minimize a cost function (Furgale'13):

$$J(\theta) := J_{feat} + J_{acc} + J_{gyro} + J_{bias_{acc}} + J_{bias_{gyro}}$$

(Feature reprojection Error) $\sum_k (a_{IMU}(t_k - t_d) - a_{cam}(t_k))^2$ $\sum_k (\omega_{IMU}(t_k - t_d) - \omega_{cam}(t_k))^2$ $\int \left\| \frac{db_{acc}}{dt}(u) \right\|^2 du$ $\int \left\| \frac{db_{gyro}}{dt}(u) \right\|^2 du$

- Unknowns: $T_{BC}, t_d, g_w, T_{WB}(t), b_{acc}(t), b_{gyro}(t)$
 - g_w = Gravity,
 - $T_{WB}(t)$ = 6-DOF trajectory of the IMU,
 - $b_{acc}(t), b_{gyro}(t)$ = 3-DOF biases of the IMU
- Continuous-time modelling using splines for $T_{WB}(t), b_{acc}(t), \dots$
- Numerical solver: Levenberg-Marquardt

Furgale et al. "Unified Temporal and Spatial Calibration for Multi-Sensor Systems", IROS'13.

<https://github.com/ethz-asl/kalibr/wiki/camera-imu-calibration>

Camera-IMU calibration - Example

➤ Software solution: Kalibr (Furgale'13).

- Generates a report after optimizing the cost function.

Residuals:

Reprojection error [px]: 0.0976 ± 0.051

Gyroscope error [rad/s]: 0.0167 ± 0.009

Accelerometer error [m/s²]: 0.0595 ± 0.031

Transformation T_{ci} (imu to cam):

```
[[ 0.99995526 -0.00934911 -0.00143776 0.00008436]
 [ 0.00936458 0.99989388 0.01115983 0.00197427]
 [ 0.00133327 -0.0111728 0.99993669 -0.05054946]
 [ 0. 0. 0. 1. ]]
```

Time shift (delay d)

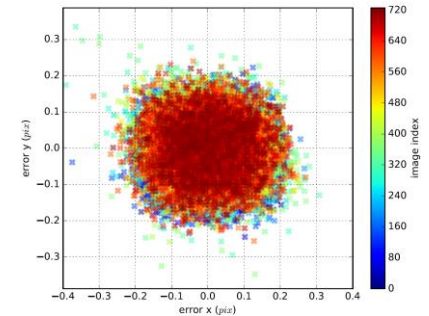
cam0 to imu0: [s] ($t_{imu} = t_{cam} + \text{shift}$)

0.00270636270255

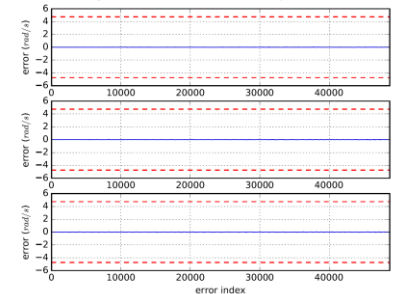
Gravity vector in target coords: [m/s²]

[0.04170719 -0.01000423 -9.80645621]

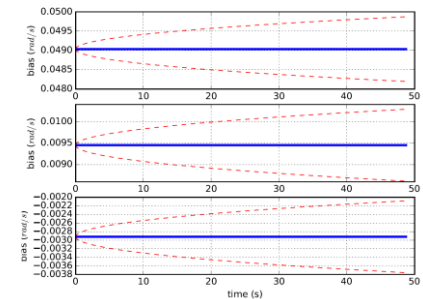
Reprojection error



Angular velocity error



Estimated gyro biases



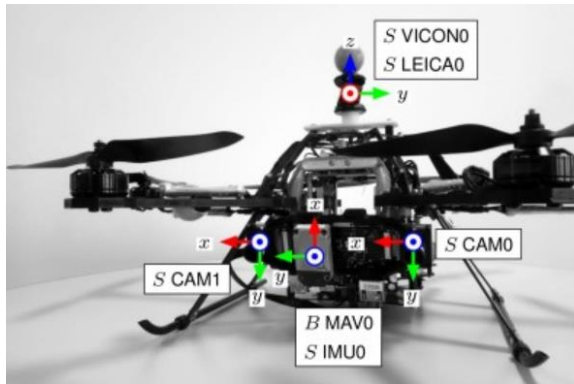
Furgale et al. "Unified Temporal and Spatial Calibration for Multi-Sensor Systems", IROS'13.

<https://github.com/ethz-asl/kalibr/wiki/camera-imu-calibration>

Popular Datasets for VIO / VI-SLAM

EuRoC [Burri'16]

MAV with synchronized IMU and stereo



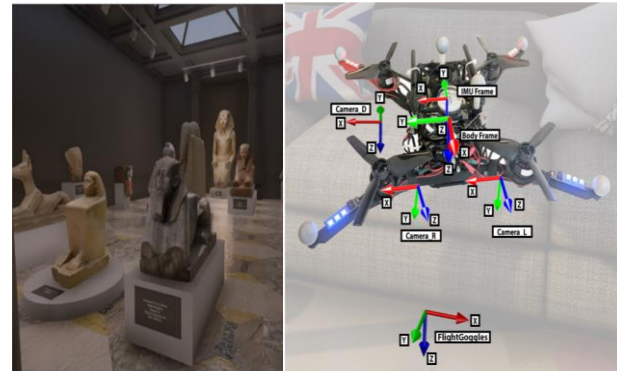
MVSEC [Zhu'18]

Events, frames, lidar, GPS, IMU from cars, drones, and motorcycles



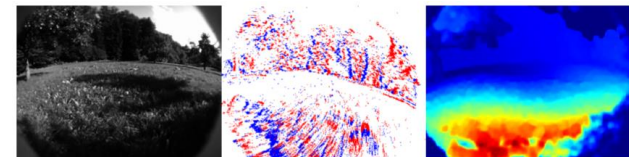
Blackbird [Antonini'18]

MAV indoor aggressive flight with rendered images and real dynamics + IMU



UZH FPV Drone Racing [Delmerico'19]

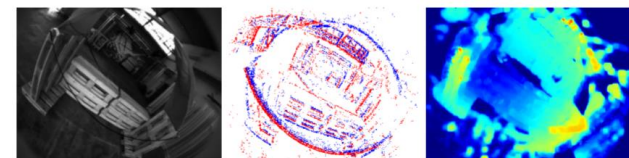
MAV aggressive flight, standard + event cameras, IMU, indoors and outdoors



(a) Outdoor sequence

(b) Events

(c) Optical flow



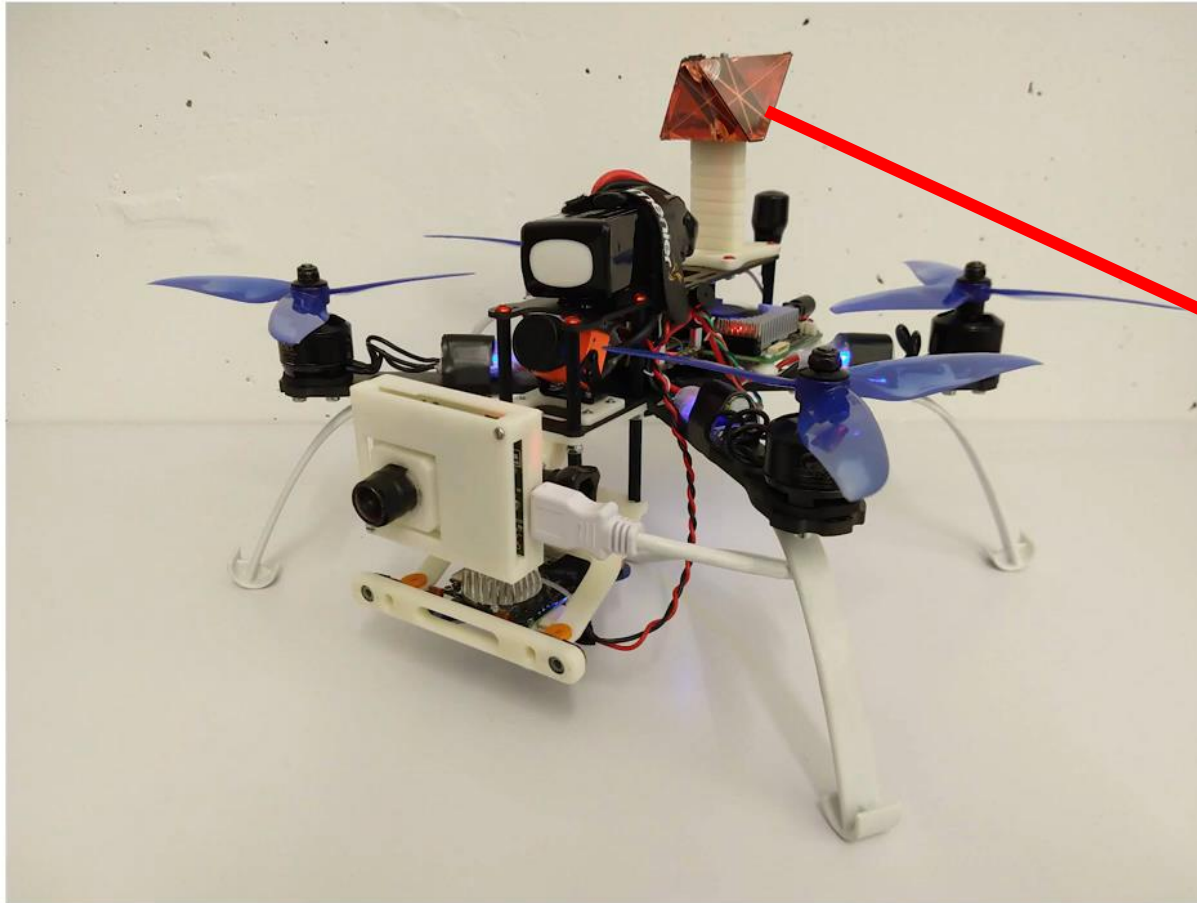
(d) Indoor sequence

(e) Events

(f) Optical flow

UZH-FPV Drone Racing Dataset

Contains data recorded by a drone flying up to over 20m/s indoors and outdoors flown by a professional pilot. Contains frames, events, IMU, and Ground Truth from a Robotic Total Station: <http://rpg.ifi.uzh.ch/uzh-fpv.html>



Delmerico et al. "Are We Ready for Autonomous Drone Racing? The UZH-FPV Drone Racing Dataset" ICRA'19
[PDF](#). [Video](#). [Datasets](#).

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[PDF](#). [Video](#). [Datasets](#).

Understanding Check

Are you able to answer the following questions?

- Why is it recommended to use an IMU for Visual Odometry?
- Why not just an IMU?
- How does a MEMS IMU work?
- What is the drift of an industrial IMU?
- What is the IMU measurement model?
- What causes the bias in an IMU?
- How do we model the bias?
- How do we integrate the acceleration to get the position formula?
- What is the definition of loosely coupled and tightly coupled visual inertial fusions?
- How can we use non-linear optimization-based approaches to solve for visual inertial fusion?