University of Zurich[UZH] | ETH zürich
Institute of Informatics – Institute of Neuroinformatics
ROBOTICS & PERCEPTION GROUP

Lecture 11
Tracking

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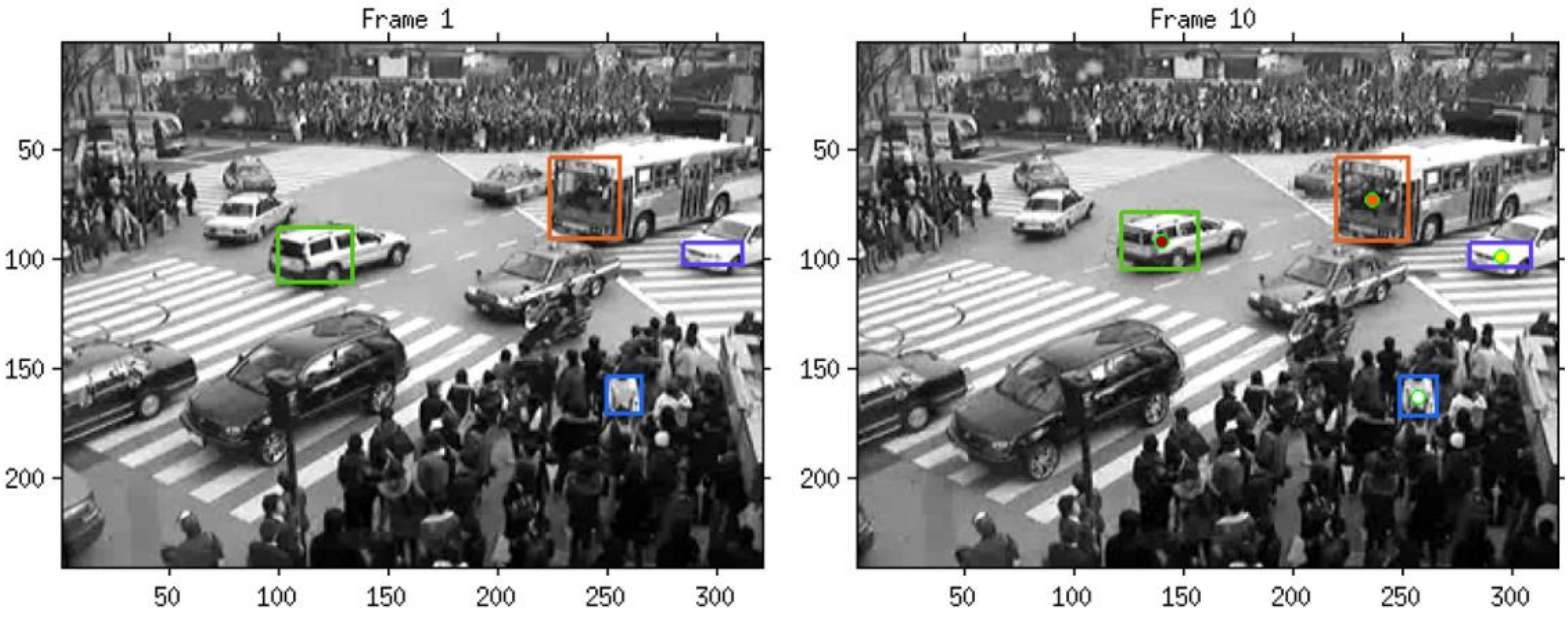
Lab Exercise 8 – Today afternoon

- Room ETH HG E 1.1 from 13:15 to 15:00
- Work description: Lucas-Kanade template tracking

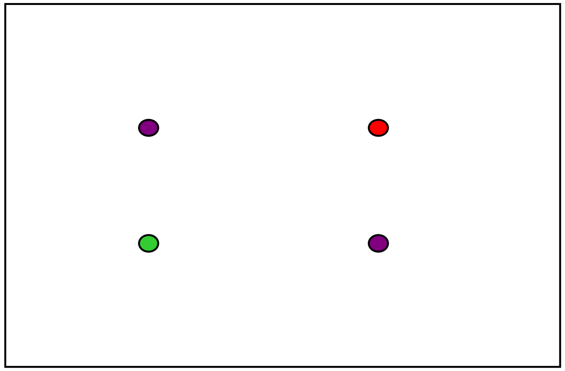


Outline

- Point tracking
- Template tracking
- Tracking by detection of local image features

Point Tracking

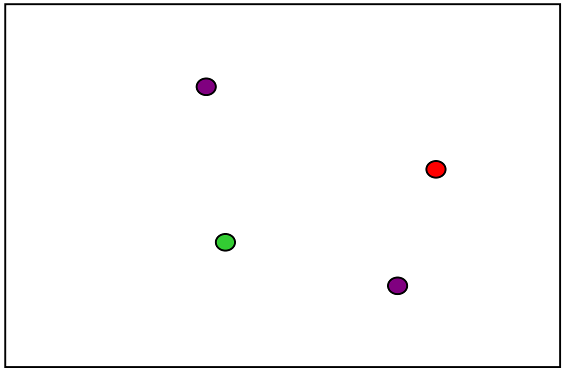
- **Problem:** given two images, estimate the motion of a pixel point from image I_0 to image I_1



$I_0(x, y)$

Point Tracking

- **Problem:** given two images, estimate the motion of a pixel point from image I_0 to image I_1

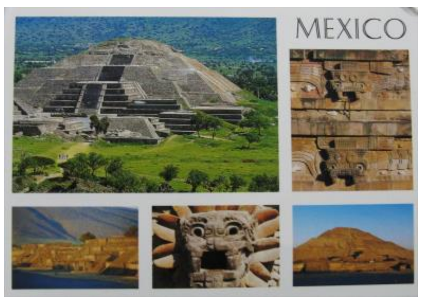


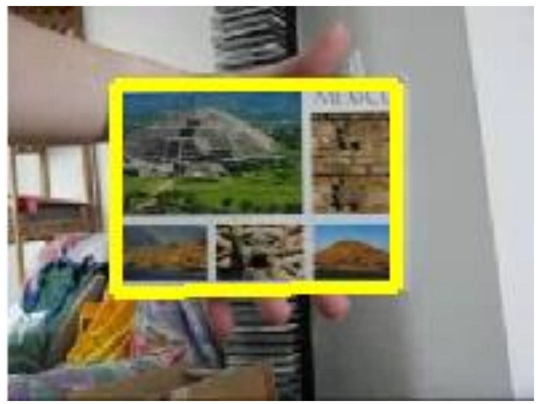
$I_1(x, y)$

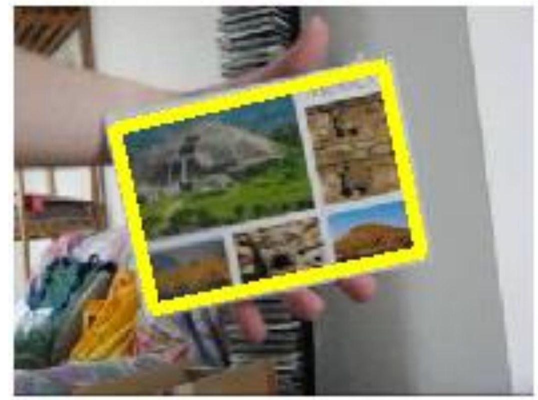
Template Tracking

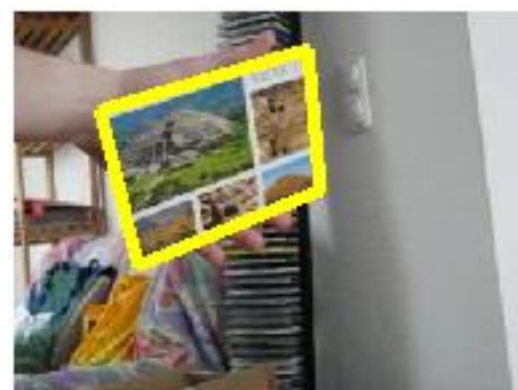
- **Problem:** given two images, estimate the *warping* that defines the *motion* and/or *the distortion* of a *template* from image I_0 to image I_1

Template image



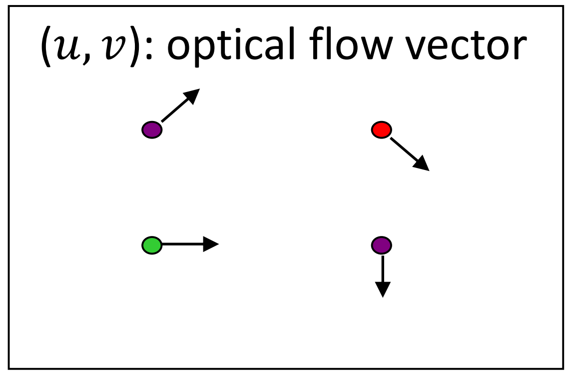






Point Tracking

- **Problem:** given two images, estimate the motion of a pixel point from image I_0 to image I_1

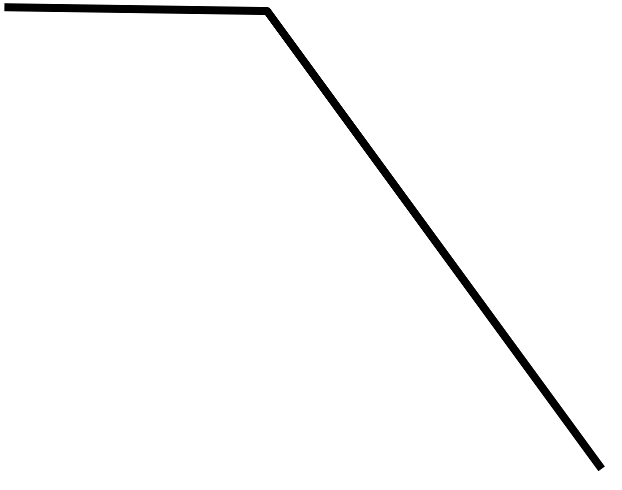


$I_0(x, y)$

- Two approaches exist, depending on the amount of motion between the frames
  - **Block-based methods**
  - **Differential methods**

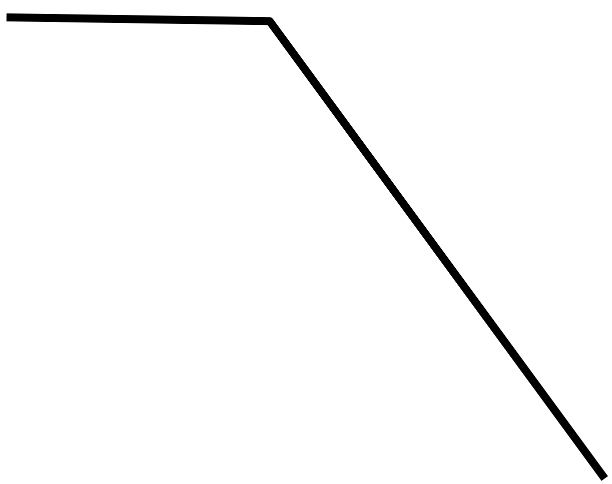
Point Tracking

- Consider the motion of the following corner



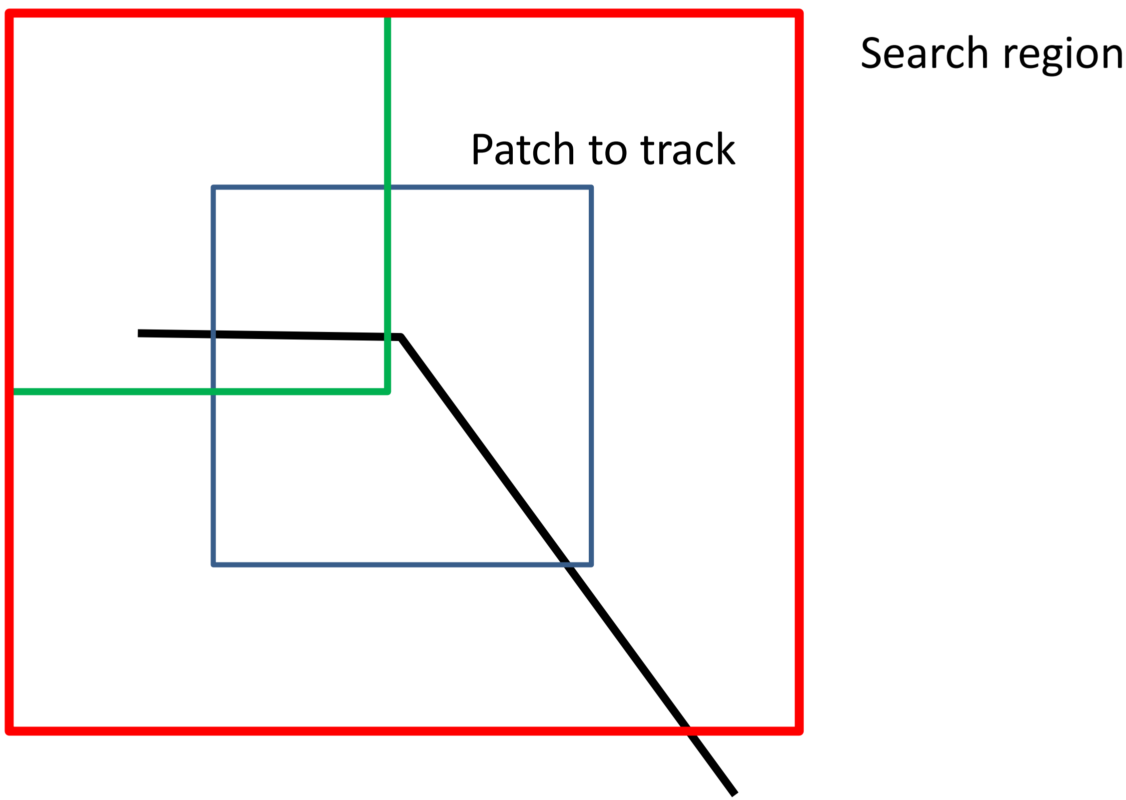
Point Tracking

- Consider the motion of the following corner



Point Tracking with Block Matching

- Search for the corresponding patch in a $D \times D$ region around the point to track.
- Use SSD, SAD, or NCC



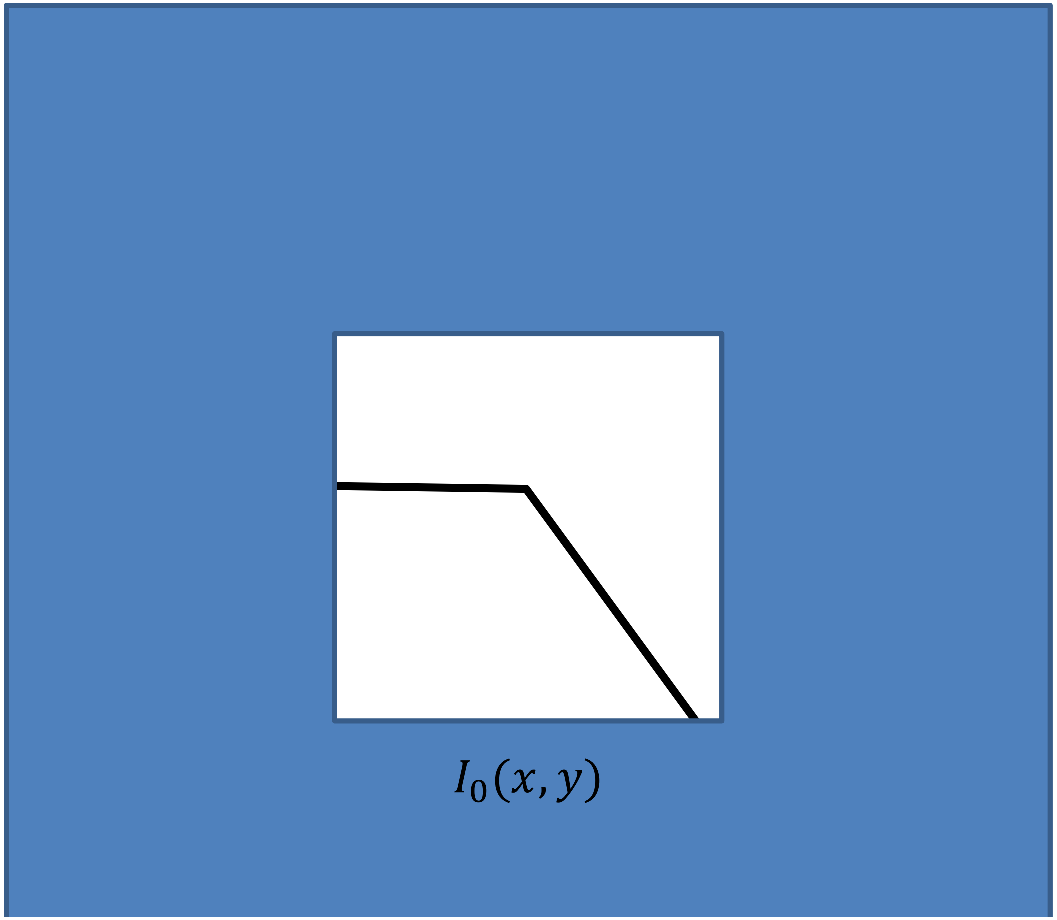
Pros and Cons of Block Matching

- Pros:
  - Works well if the motion is large
- Cons
  - Can become computationally demanding if the motion is large

- Can the “search” be implemented in a smart way if the motion is “small”?
  - Yes, use Differential methods

Point Tracking with Differential Methods

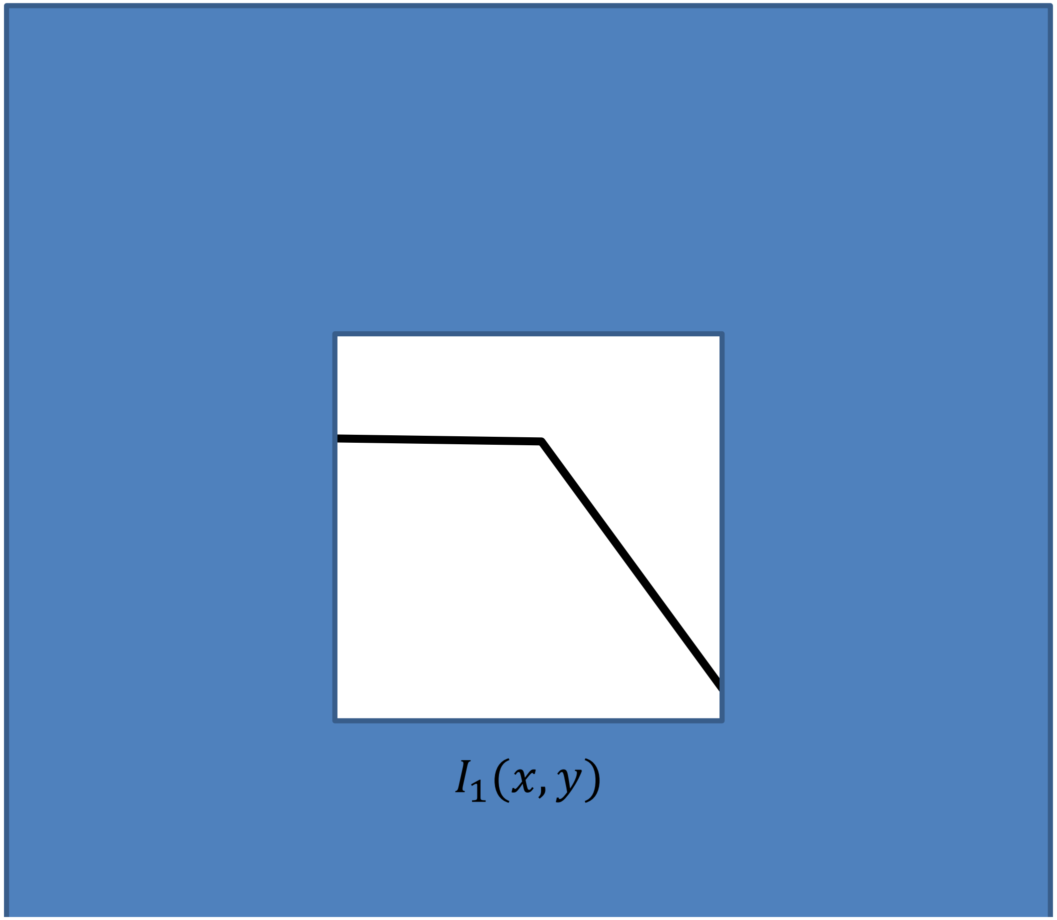
- Looks at the local brightness changes at the **same** location. **No patch shift** is performed!



B. D. Lucas and T. Kanade, *An iterative image registration technique with an application to stereo vision.* Proceedings of Imaging Understanding Workshop, 1981

Point Tracking with Differential Methods

- Looks at the local brightness changes at the **same** location. **No patch shift** is performed!



B. D. Lucas and T. Kanade, *An iterative image registration technique with an application to stereo vision.* Proceedings of Imaging Understanding Workshop, 1981

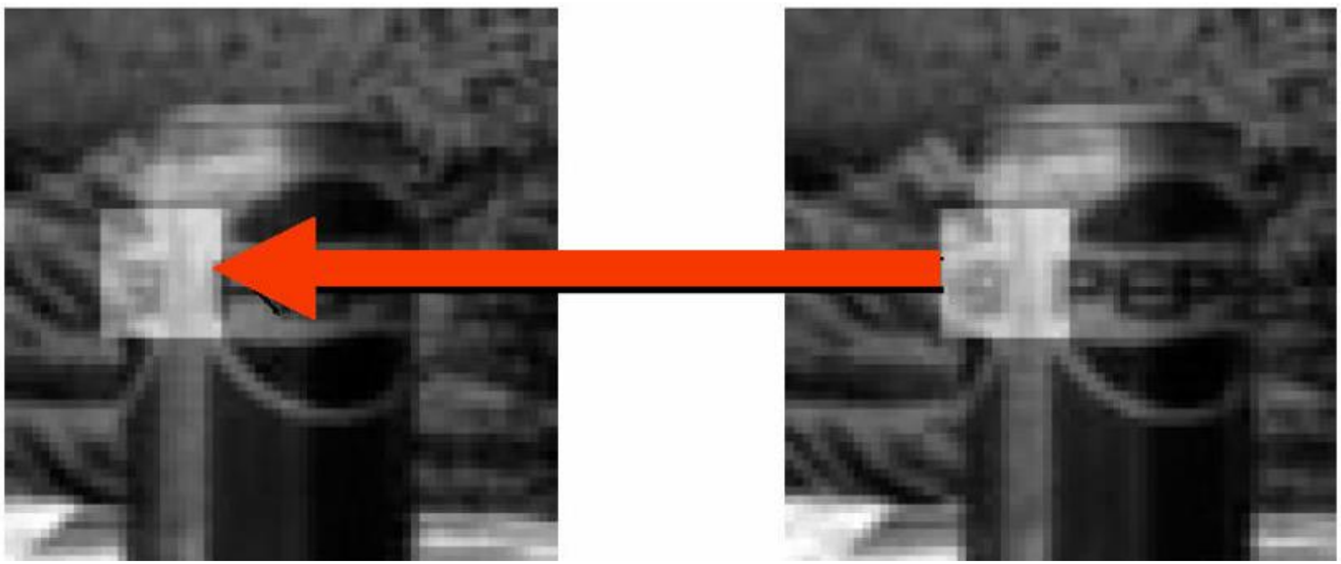
Point Tracking with Differential Methods

Assumptions:

1. Brightness constancy
2. Temporal consistency
3. Spatial coherency

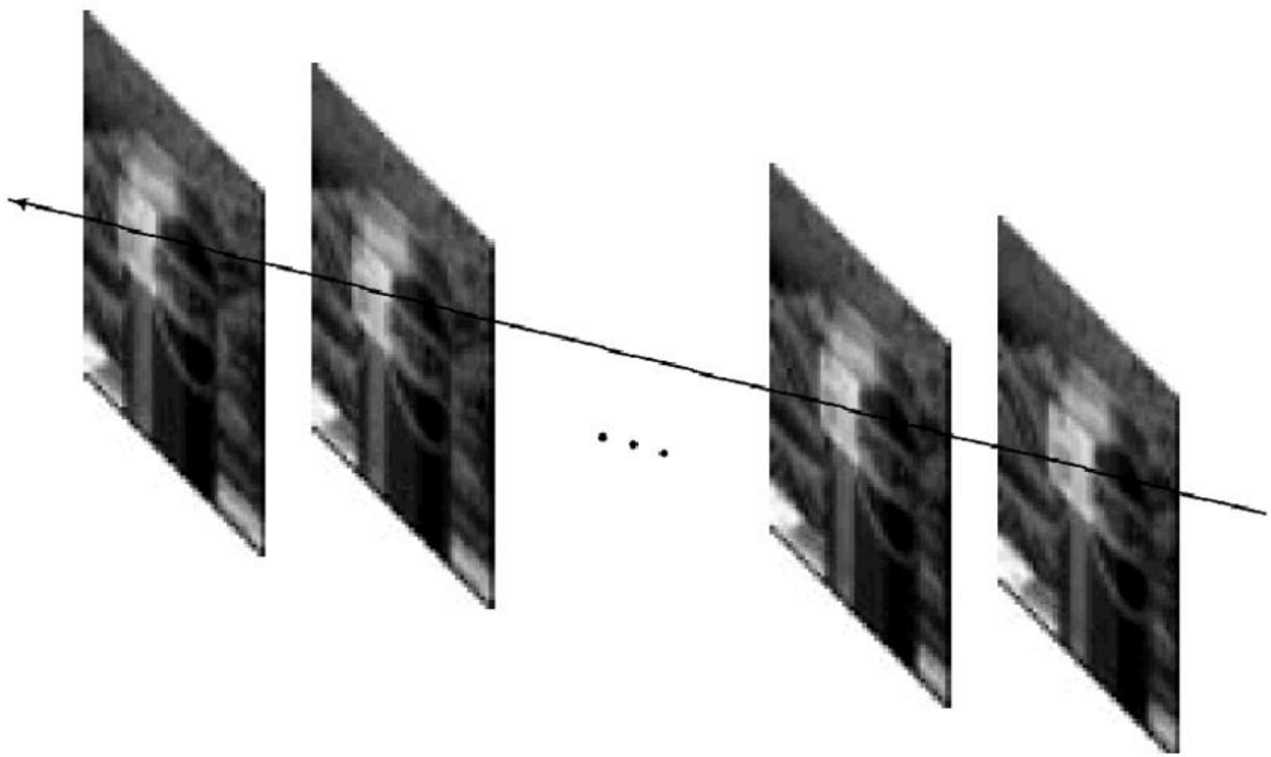
Brightness Constancy

- The intensity of the pixels around the point to track in image I_0 should be the same of its corresponding pixels in image I_1



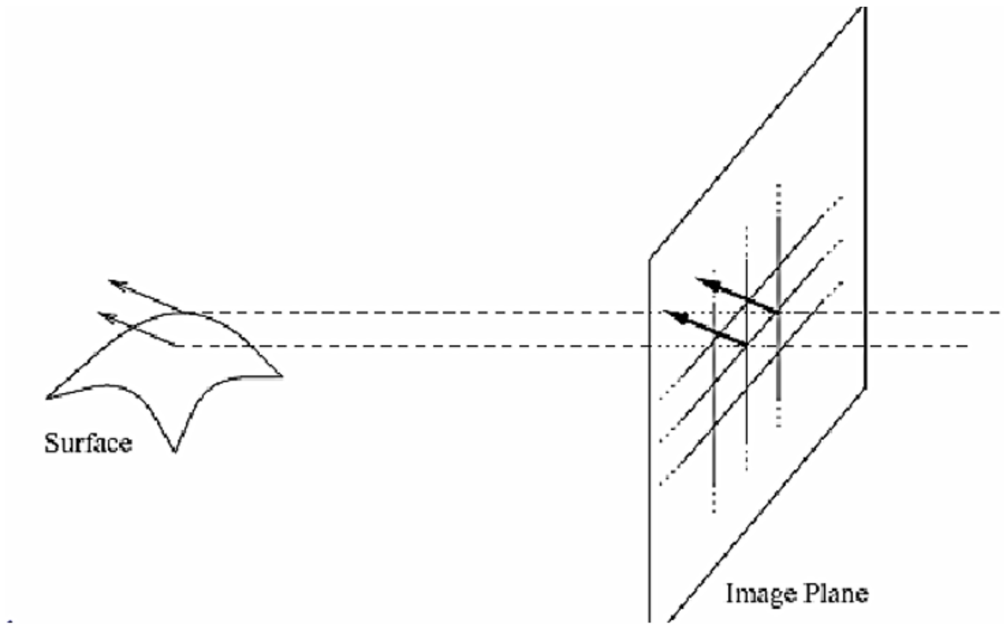
Temporal Consistency (Short Base-Line)

- The motion between the two frames must be small (1-2 pixels at the most)



Spatial Coherency

- Neighboring pixels belonging to the same surface and therefore undergo similar motion



Point Tracking with Differential Methods: Approach

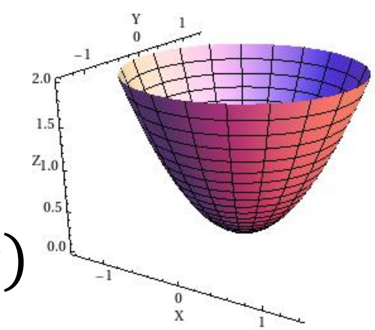
- We want to find the motion vector (u, v) that minimizes the Sum of Squared Differences (SSD):

$$SSD = \sum (I_0(x, y) - I_1(x + u, y + v))^2$$

$$\cong \sum (I_0(x, y) - I_1(x, y) - I_x u - I_y v)^2$$

$$= \sum (\Delta I - I_x u - I_y v)^2$$

This is a simple quadratic function in two variables (u, v)



Point Tracking with Differential Methods: Approach

$$E = SSD = \sum(\Delta I - I_x u - I_y v)^2$$

- To minimize the E, we differentiate E with respect to (u, v) and equate it to zero

$$\frac{\partial E}{\partial u} = 0\,,\ \frac{\partial E}{\partial v} = 0$$

$$\frac{\partial E}{\partial u} = 0 \ \Rightarrow\ -2\sum I_x(\Delta I - I_x u - I_y v) \ = 0$$

$$\frac{\partial E}{\partial v} = 0 \ \Rightarrow\ -2\sum I_y\left(\Delta I - I_x u - I_y v\right) \ = 0$$

Point Tracking with Differential Methods: Approach

$$\frac{\partial E}{\partial u} = 0 \;\Rightarrow\; -2\sum I_x(\Delta I - I_x u - I_y v) \;\; = 0$$

$$\frac{\partial E}{\partial v} = 0 \;\Rightarrow\; -2\sum I_y\left(\Delta I - I_x u - I_y v\right) \;\; = 0$$

Point Tracking with Differential Methods: Approach

$$\sum I_x(\Delta I - I_x u - I_y v) = 0$$

$$\sum I_y(\Delta I - I_x u - I_y v) = 0$$

- Linear system of two equations in two unknowns
- We can write them in matrix form:

Notice that these are NOT matrix products but **pixel-wise** products!

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x \Delta I \\ \sum I_y \Delta I \end{bmatrix} \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}^{-1} \begin{bmatrix} \sum I_x \Delta I \\ \sum I_y \Delta I \end{bmatrix}$$

M matrix

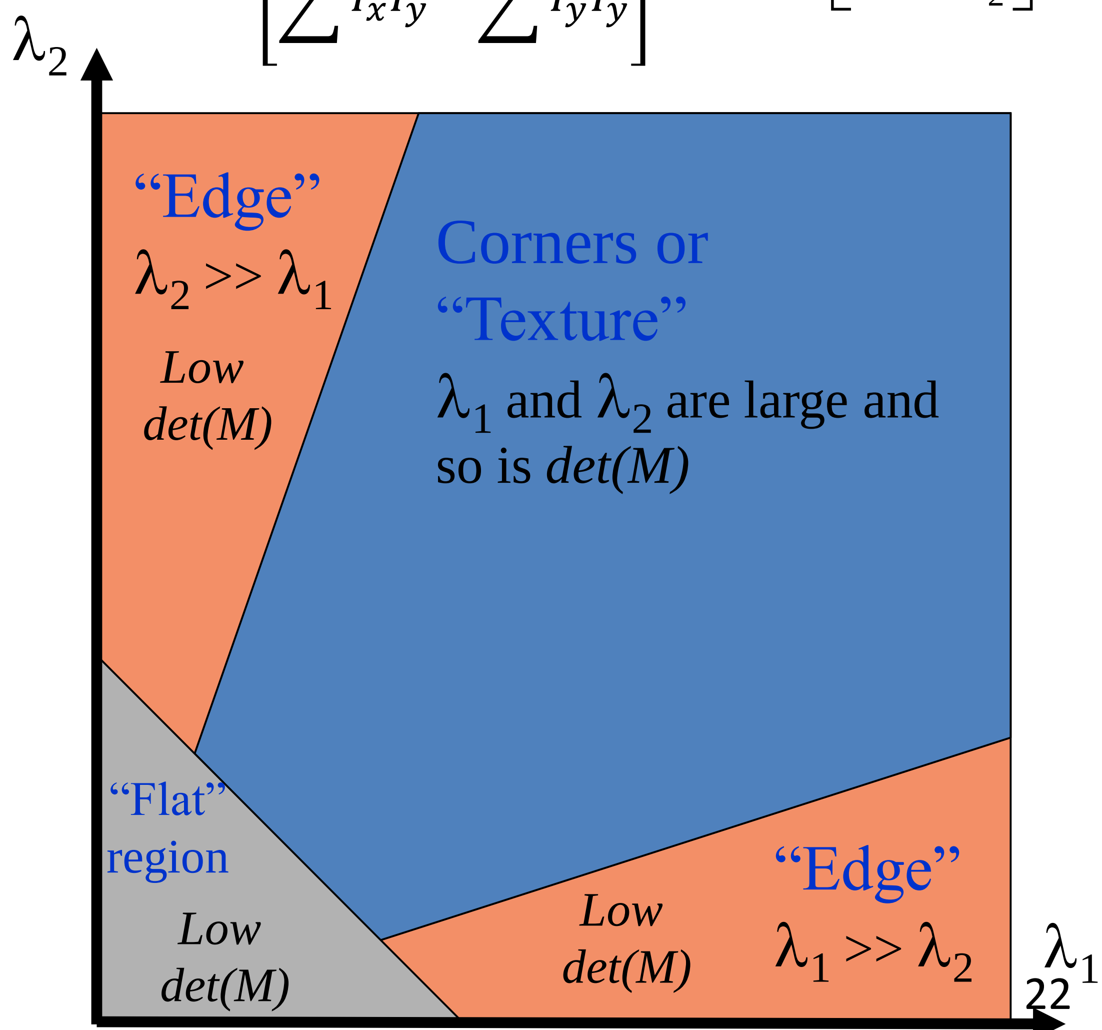
Haven't we seen this matrix already?

Recall Harris detector!

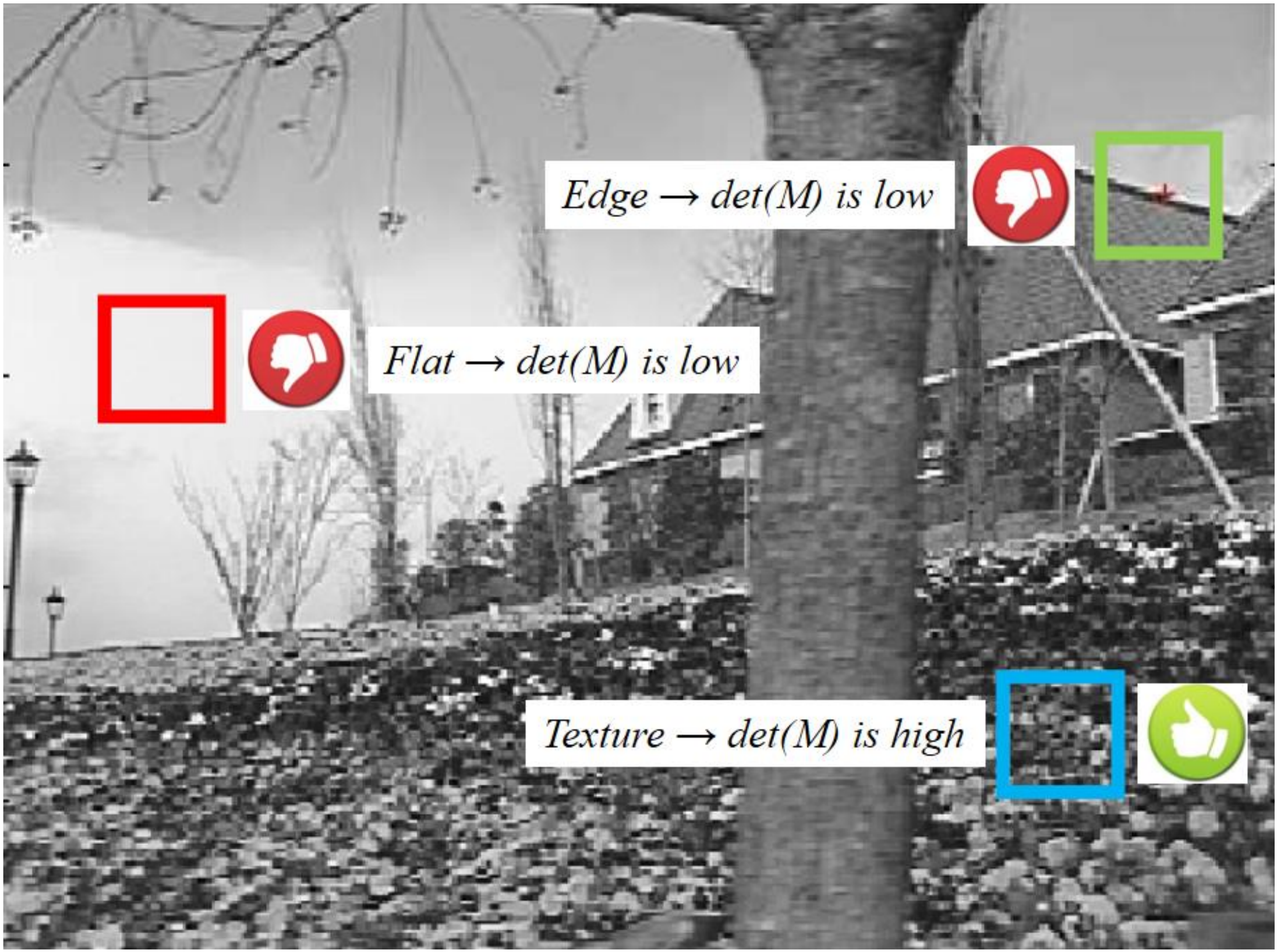
For M to be invertible, its determinant has to be non zero

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

- In practice, *det(M)* should be non zero, which means that its eigenvalues should be large (i.e., not a flat region, not an edge) -> in practice, it **should be a corner or more generally contain texture**!



What are good features to track?

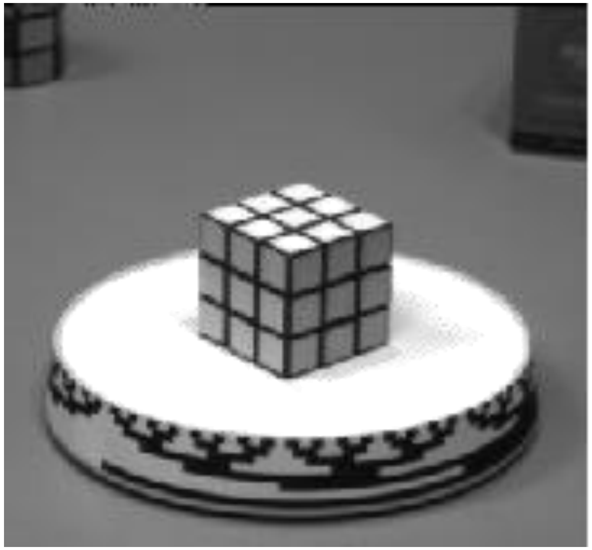
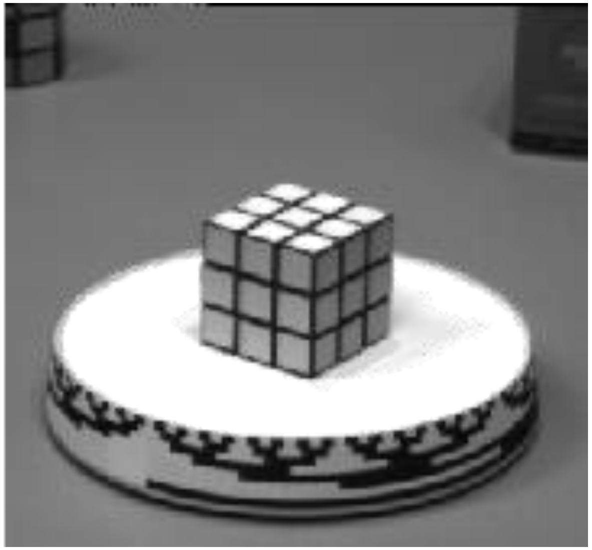
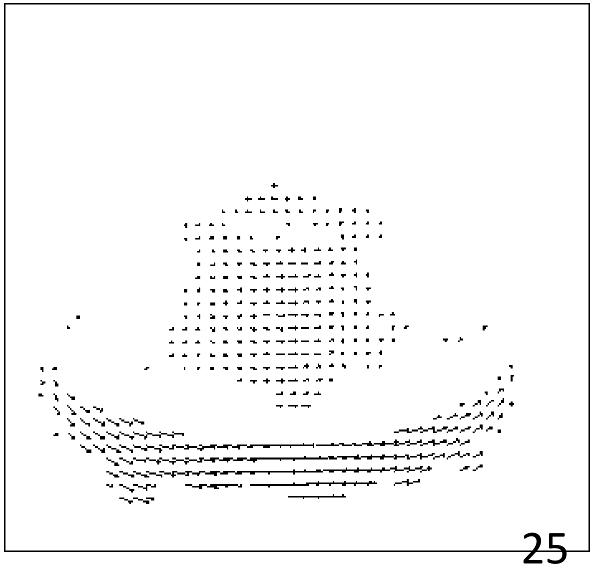


Application to Corner Tracking

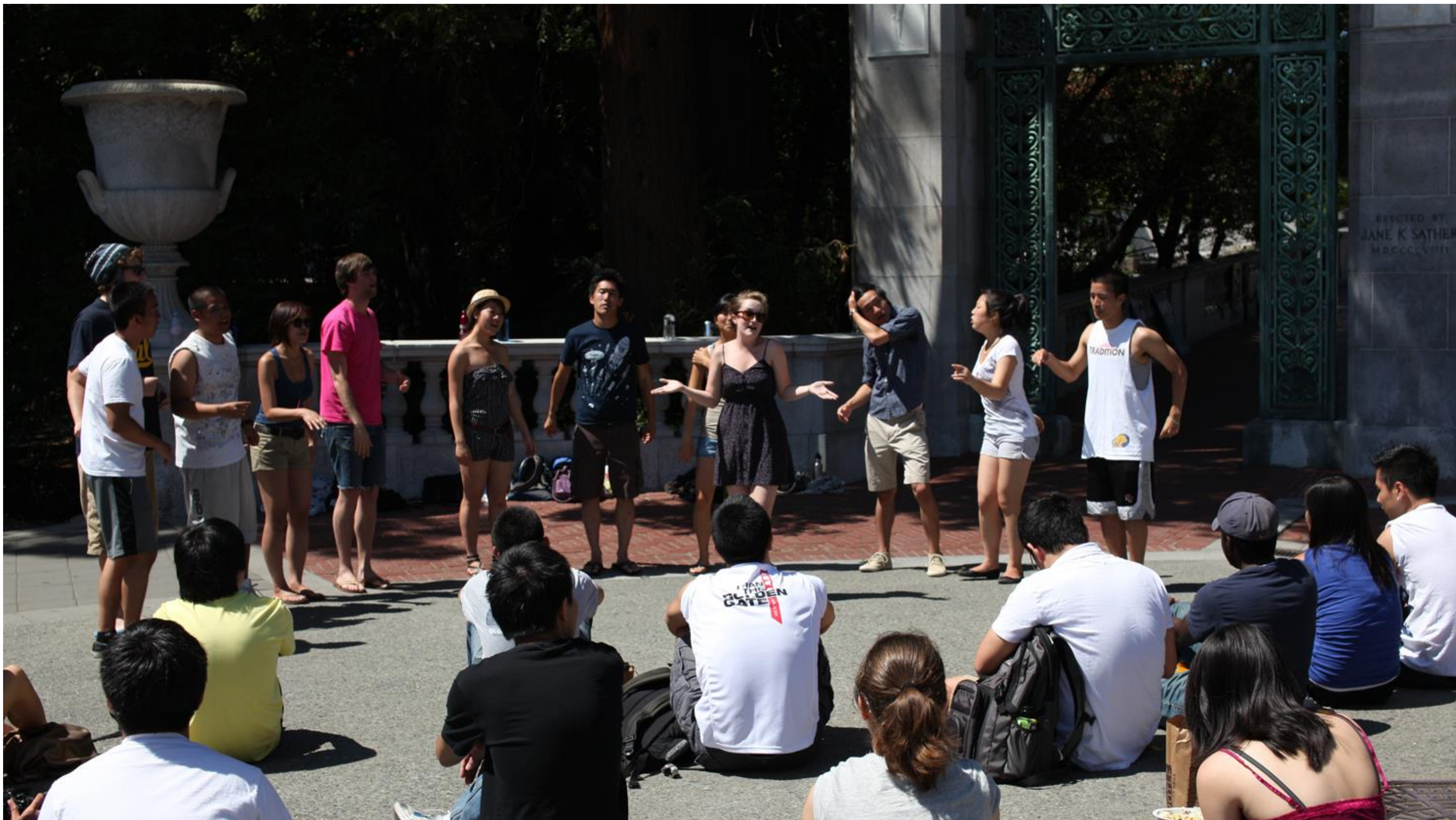
Color encodes motion direction

Application of Differential Methods: Optical Flow

- **Optical flow** or **optic flow** is the pattern of apparent motion of objects in a visual scene caused by the relative motion between the observer (an eye or a camera) and the scene
- Tracks the motion of every pixels (or a grid of pixels) between two consecutive frames
- For each pixel, a **motion vector** is computed:
  - Vector **direction** represents motion direction
  - Vector **length** represents the amount of movement

Optical Flow



[Tao et al., Eurographics 2012]

Optical Flow

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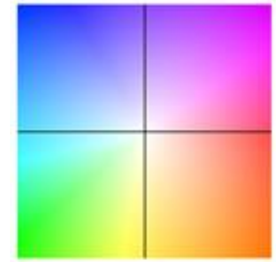
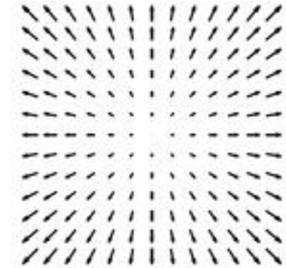
Optical Flow



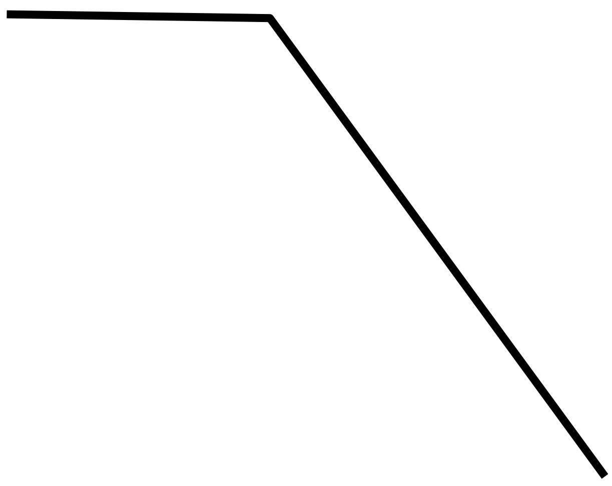


[Tao et al., Eurographics 2012]

Optical Flow example

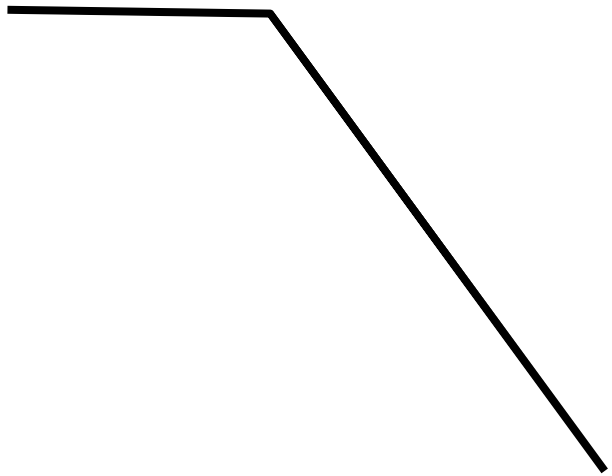
Aperture Problem

- Consider the motion of the following corner



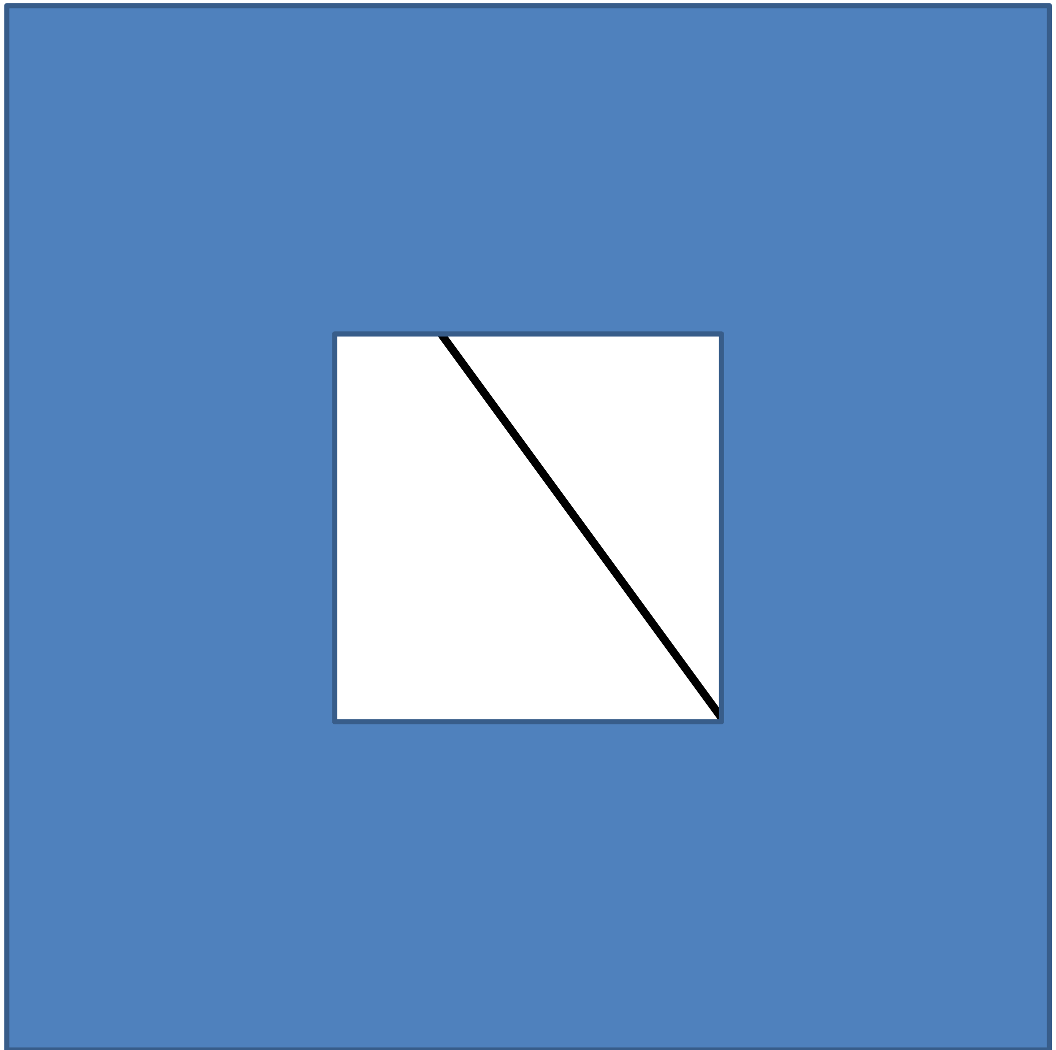
Aperture Problem

- Consider the motion of the following corner



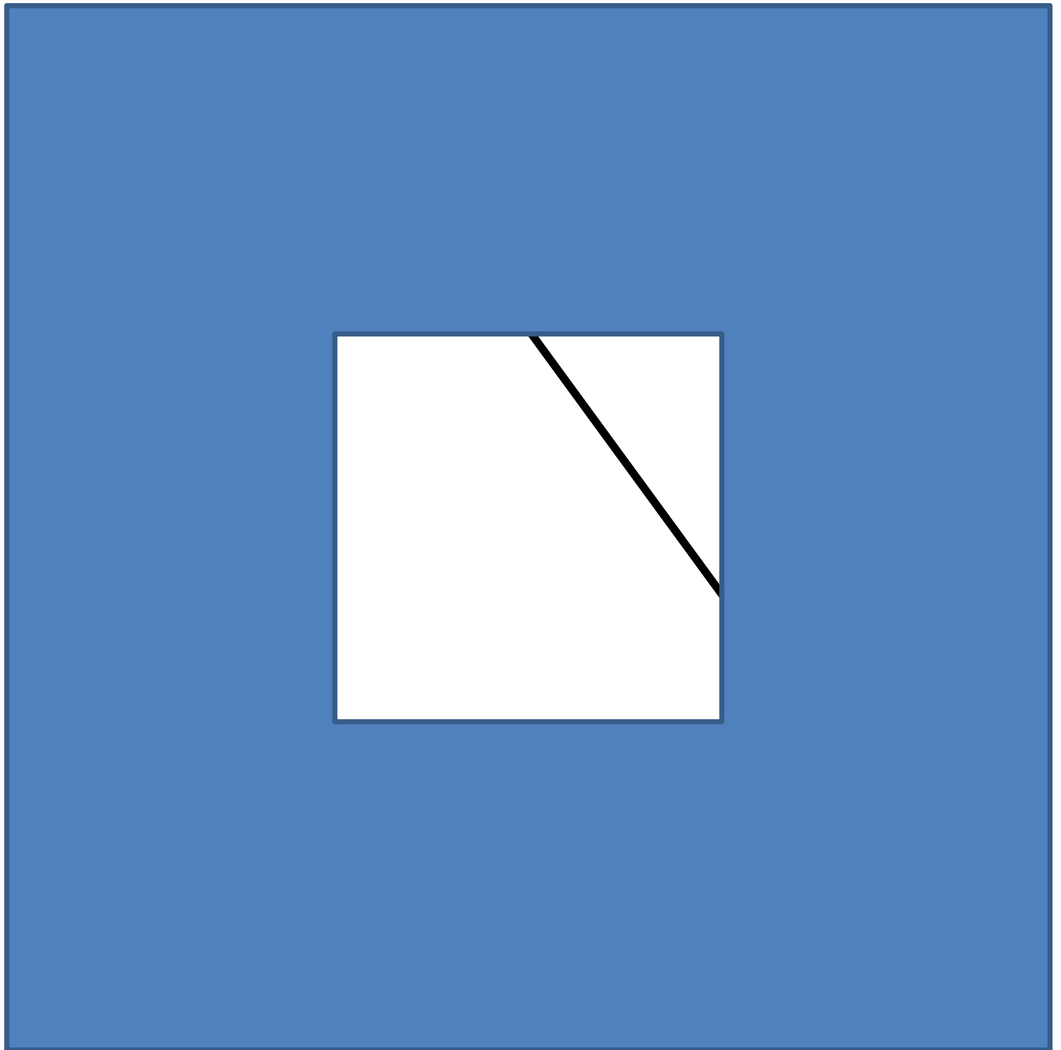
Aperture Problem

- Now, look at the local brightness changes **through a small aperture**



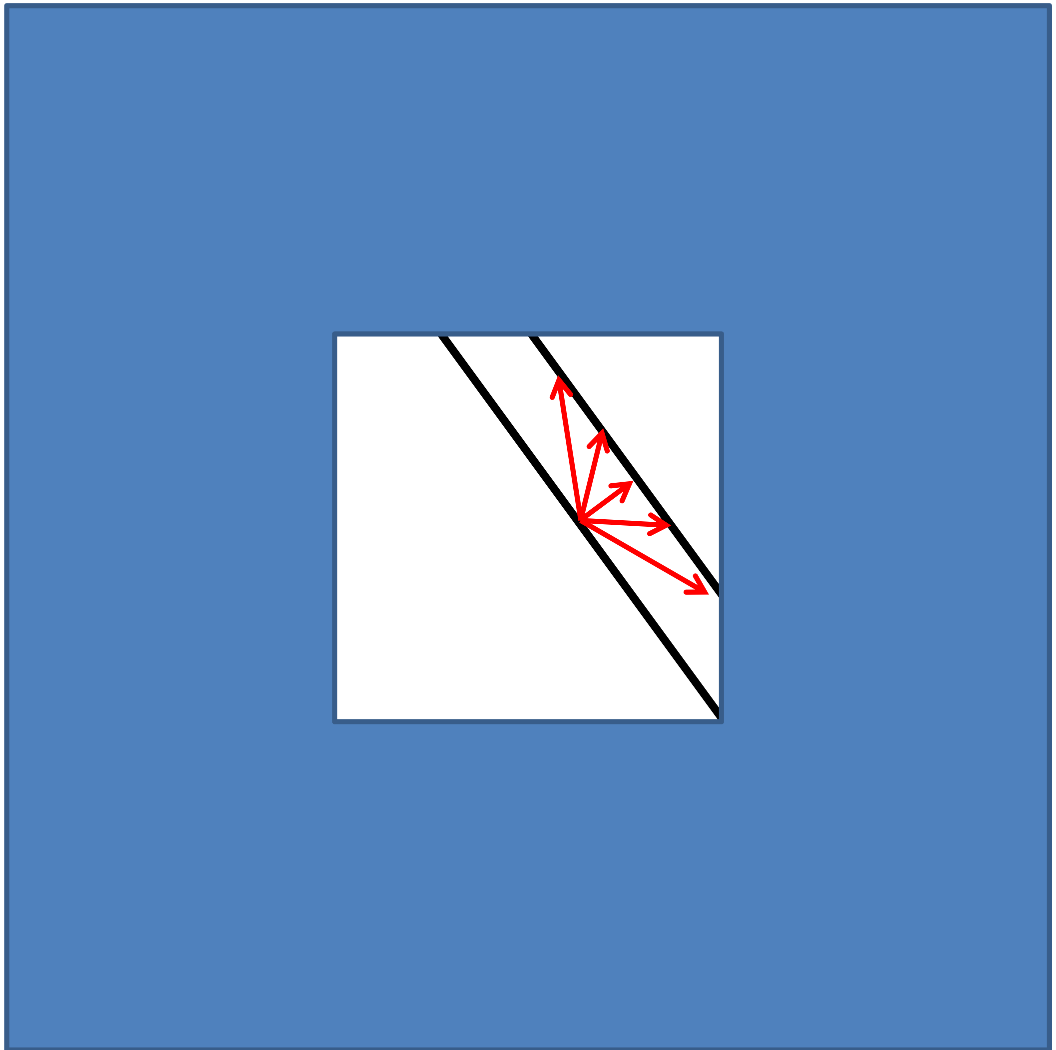
Aperture Problem

- Now, look at the local brightness changes **through a small aperture**



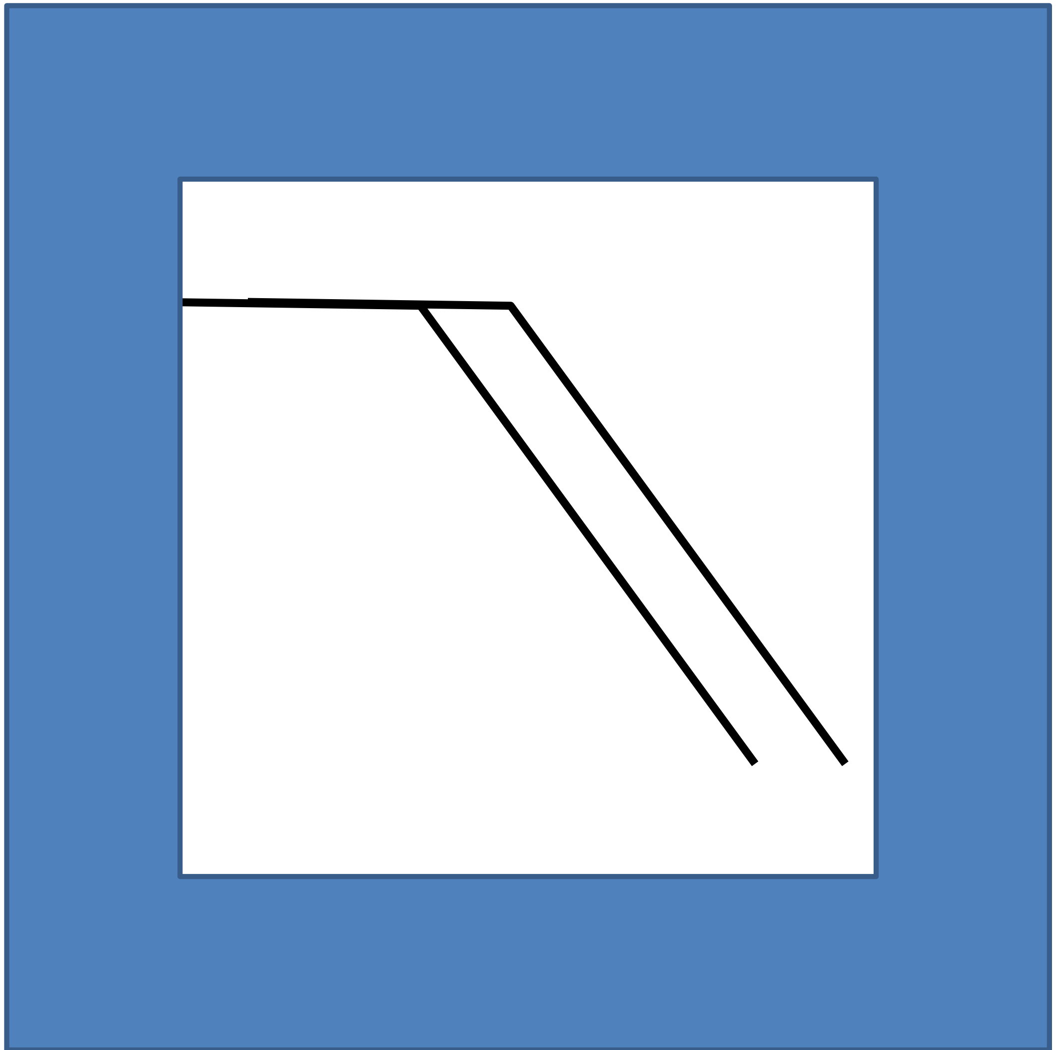
Aperture Problem

- Now, look at the local brightness changes **through a small aperture**
- We **cannot always determine** the motion direction -> **Infinite motion solutions** may exist!
- Solution?



Aperture Problem

- Now, look at the local brightness changes **through a small aperture**
- We **cannot always determine** the motion direction -> **Infinite motion solutions** may exist!
- Solution? Increase aperture size!



Block-based vs. Differential methods

- **Block-based methods:** search for the corresponding patch in a neighborhood of the point to be tracked. The search region is usually a square of $D \times D$ pixels.

  Robust to large motions

  Can be **computationally expensive** ($D \times D$ validations need to be made for a single point to track)

- **Differential methods:**

  Works only for **small motions** (e.g., high frame rate). For larger motion, multi-scale implementations are used but are more expensive

  Much more **efficient** than block-based methods. Thus, can be used to track the motion of every pixel in the image (i.e., optical flow). It avoids searching in the neighborhood of the point by analyzing the **local intensity changes** (i.e., differences) of an image patch at a **specific location** (i.e., no search is performed).

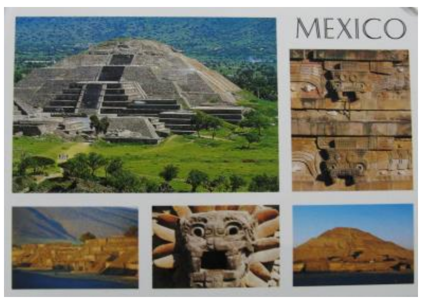
Outline

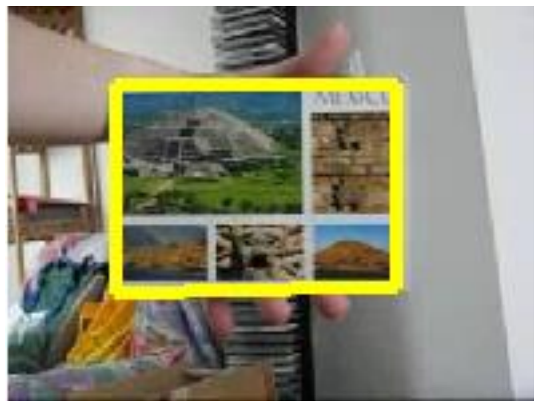
- Point tracking
- Template tracking
- Tracking by detection of local image features

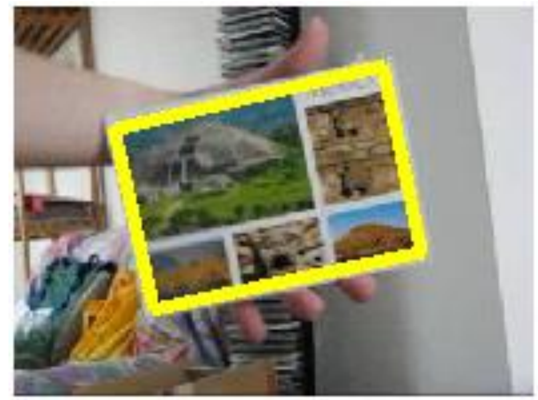
Template tracking

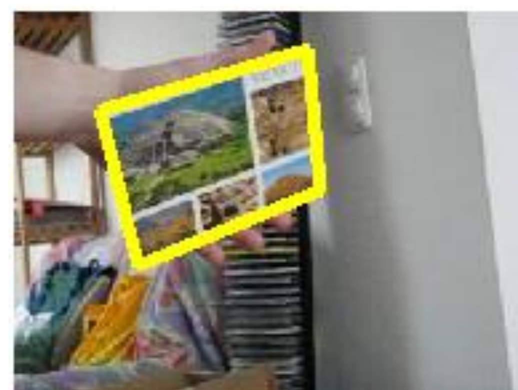
Definition: follow a template image in a video sequence by estimating the warp

Template image

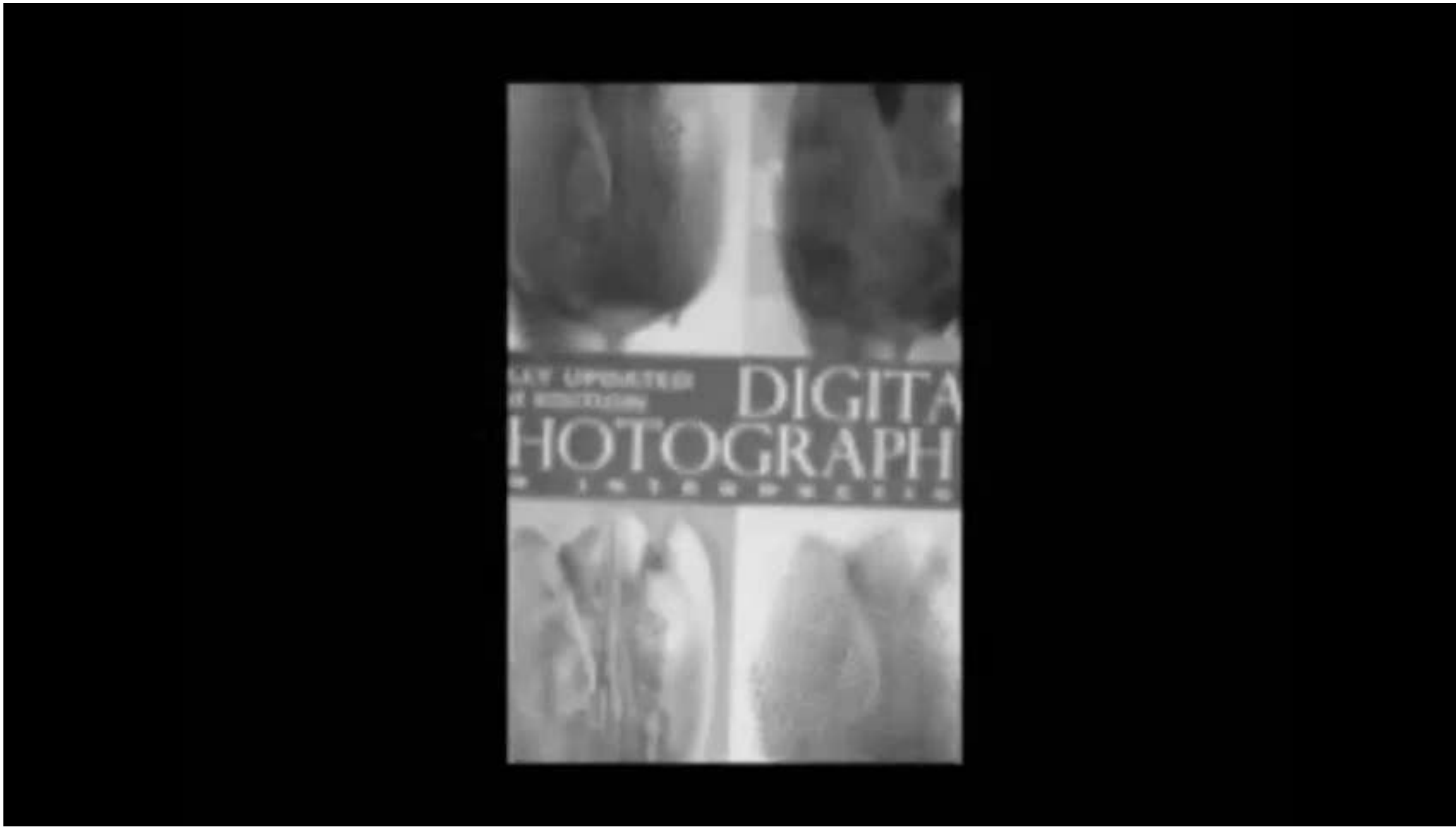








The Lucas-Kanade tracker

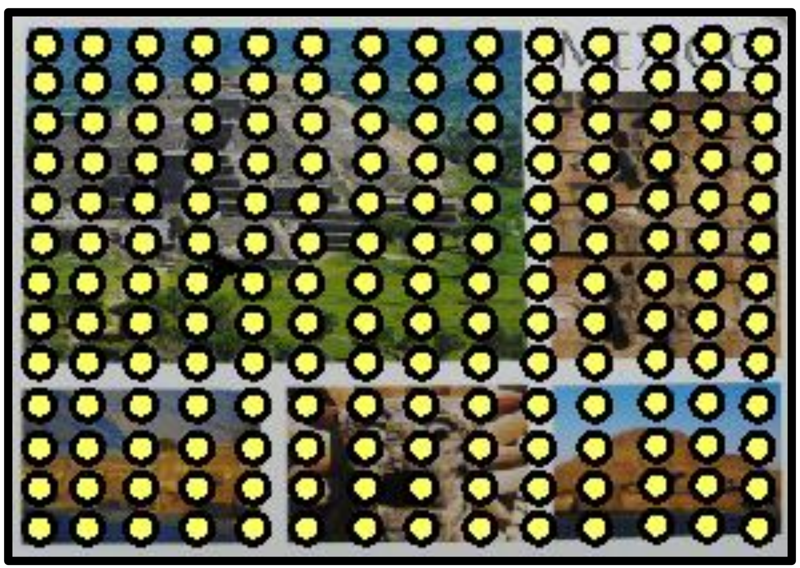


B. D. Lucas and T. Kanade (1981), *An iterative image registration technique with an application to stereo vision.* Proceedings of Imaging Understanding Workshop, pages 121--130

Template Warping

- Given the template image $T(\mathbf{x})$
- Take all pixels from the template image $T(\mathbf{x})$ and warp them using the function $W(\mathbf{x}, \mathbf{p})$ parameterized in terms of parameters $\mathbf{p}$

Template image

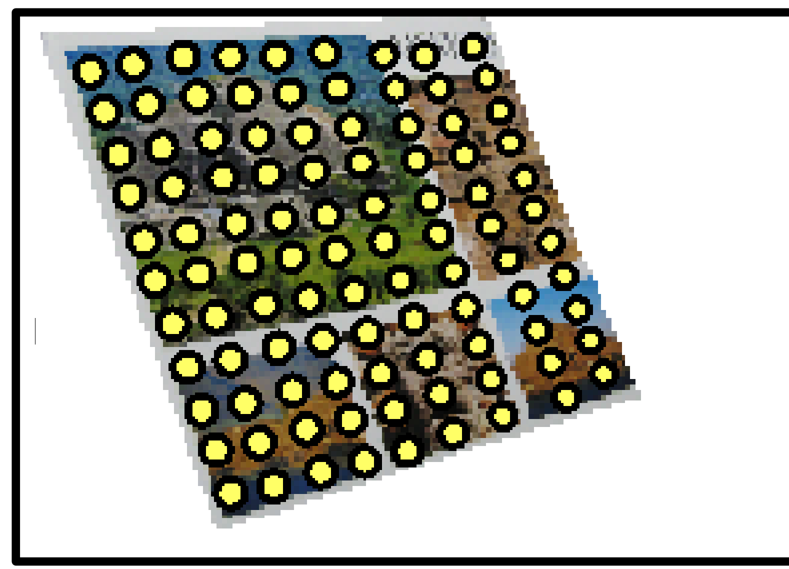


$T(\mathbf{x})$

warp

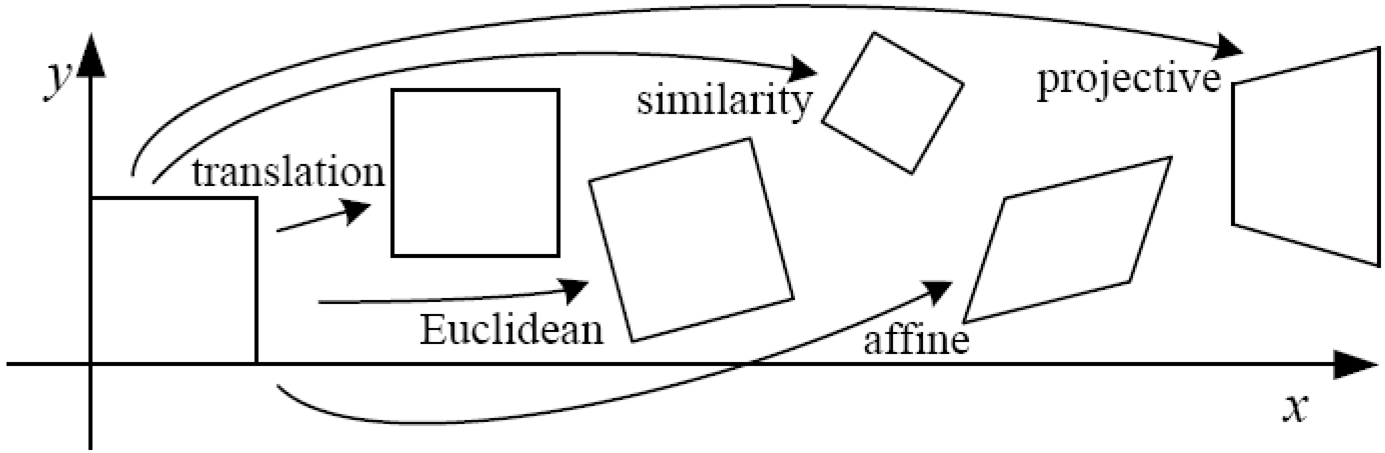
$W(\mathbf{x}, \mathbf{p})$

Current image



$I(W(\mathbf{x}, \mathbf{p}))$

Common 2D Transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\left[\, \boldsymbol{I} \mid \boldsymbol{t} \,\right]_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\left[\, \boldsymbol{R} \mid \boldsymbol{t} \,\right]_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\left[\, s\boldsymbol{R} \mid \boldsymbol{t} \,\right]_{2\times 3}$	4	angles $+\cdots$	
affine	$\left[\, \boldsymbol{A} \,\right]_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\left[\, \tilde{\boldsymbol{H}} \,\right]_{3\times 3}$	8	straight lines	

Common 2D Transformations

- Translation

$$x' = x + a_1$$
$$y' = y + a_2$$

- Euclidean

$$x' = xcos(a_3) - ysin(a_3) + a_1$$
$$y' = xsin(a_3) + ycos(a_3) + a_2$$

- Affine

$$x' = a_1x + a_3y + a_5$$
$$y' = a_2x + a_4y + a_6$$

- Projective (homography)

$$x' = \frac{a_1x + a_2y + a_3}{a_7x + a_8y + 1}$$

$$y' = \frac{a_4x + a_5y + a_6}{a_7x + a_8y + 1}$$

2D Warping

We denote the transformation $\mathrm{W}(\mathbf{x}, \mathbf{p})$ and $\mathbf{p}$ the set of parameters $p = (a_1, a_2, \dots, a_n)$

- Translation $$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} x + a_1 \\ y + a_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

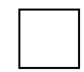
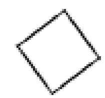
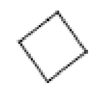
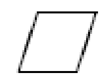
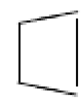
Homogeneous coordinates

- Euclidean $$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} x cos(a_3) - y sin(a_3) + a_1 \\ x sin(a_3) + y cos(a_3) + a_2 \end{bmatrix} = \begin{bmatrix} c a_3 & -s a_3 & a_1 \\ s a_3 & c a_3 & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Affine $$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} a_1 x + a_3 y + a_5 \\ a_2 x + a_4 y + a_6 \end{bmatrix} = \begin{bmatrix} a_1 & a_3 & a_5 \\ a_2 & a_4 & a_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Projective $$W(\tilde{\mathbf{x}}, \mathbf{p}) = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Summary of displacement models (2D transformations)

Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} I \mid t \end{bmatrix}_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\begin{bmatrix} R \mid t \end{bmatrix}_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{2\times 3}$	4	angles $+\cdots$	
affine	$\begin{bmatrix} A \end{bmatrix}_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3\times 3}$	8	straight lines	

$$W(\mathbf{x},\mathbf{p}) = \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$W(\mathbf{x},\mathbf{p}) = \begin{bmatrix} ca_3 & -sa_3 & a_1 \\ sa_3 & ca_3 & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$W(\mathbf{x},\mathbf{p}) = \lambda \begin{bmatrix} ca_3 & -sa_3 & a_1 \\ sa_3 & ca_3 & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$W(\mathbf{x},\mathbf{p}) = \begin{bmatrix} a_1 & a_3 & a_5 \\ a_2 & a_4 & a_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$W(\tilde{\mathbf{x}},\mathbf{p}) = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Derivative and gradient

- Function: $f(x)$
- Derivative: $f'(x) = \frac{df}{dx}$, where x is a scalar
- Function: $f(x_1, x_2, \dots, x_n)$
- Gradient: $\nabla f(x_1, x_2, \dots, x_n) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}\right)$

Jacobian

- $F(x_1, x_2, \dots, x_n) = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n) \\ \vdots \\ f_m(x_1, x_2, \dots, x_n) \end{bmatrix}$

Vector-valued function

Derivative?

$$J(F) = \nabla F = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}, \dots, \frac{\partial f_1}{\partial x_n} \\ \vdots \\ \frac{\partial f_m}{\partial x_1}, \dots, \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Carl Gustav Jacob (1804-1851)

Displacement-model Jacobians ∇W_p

$$p = (a_1, a_2, \ldots, a_n)$$

- Translation

$$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} x + a_1 \\ y + a_2 \end{bmatrix} \qquad \nabla W_p = \begin{bmatrix} \frac{\partial W_1}{\partial a_1} & \frac{\partial W_1}{\partial a_2} \\ \frac{\partial W_2}{\partial a_1} & \frac{\partial W_2}{\partial a_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Euclidean

$$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} xcos(a_3) - ysin(a_3) + a_1 \\ xsin(a_3) + ycos(a_3) + a_2 \end{bmatrix} \qquad \nabla W_p = \begin{bmatrix} 1 & 0 & -xsin(a_3) - ycos(a_3) \\ 0 & 1 & xcos(a_3) - ysin(a_3) \end{bmatrix}$$

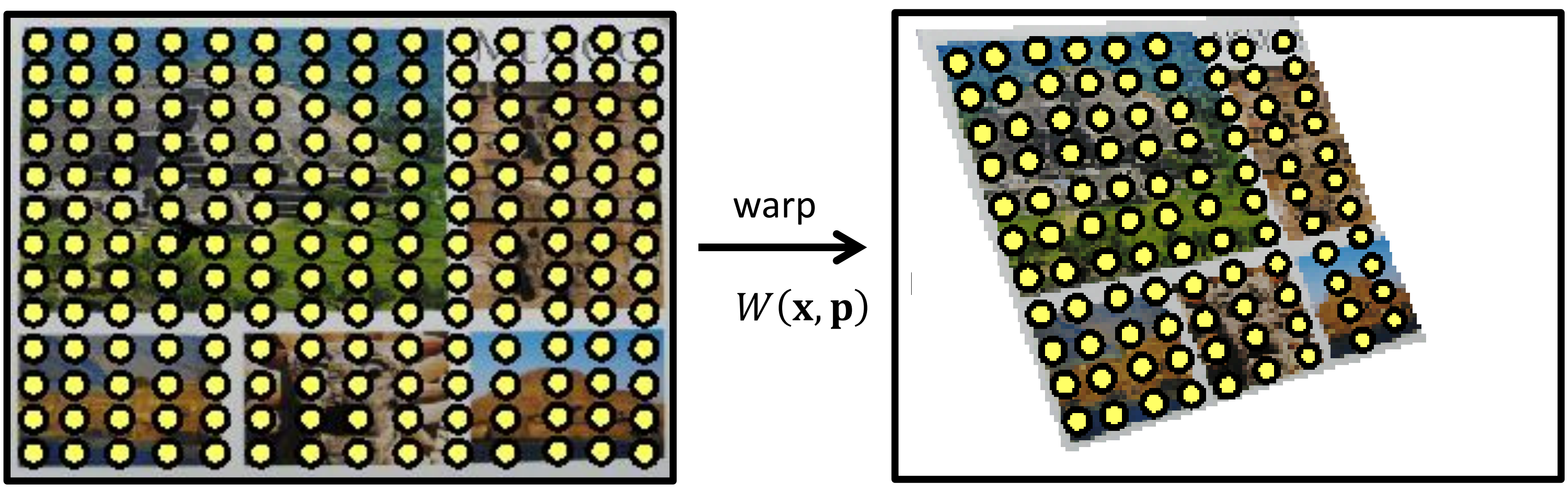
- Affine

$$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} a_1 x + a_3 y + a_5 \\ a_2 x + a_4 y + a_6 \end{bmatrix} \qquad \nabla W_p = \begin{bmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{bmatrix}$$

Template Warping

- Given the template image $T(\mathbf{x})$
- Take all pixels from the template image $T(\mathbf{x})$ and warp them using the function $W(\mathbf{x}, \mathbf{p})$ parameterized in terms of parameters $\mathbf{p}$

Template image **Current image**



$T(\mathbf{x})$ $I(W(\mathbf{x}, \mathbf{p}))$

Template Tracking: Problem Formulation

- The goal of template-based tracking is to find the set of warp parameters **p** such that:

$$I\big(W(\mathbf{x},\mathbf{p})\big) = T(\mathbf{x})$$

This is solved by determining **p** that minimizes the Sum of Squared Differences

$$E = SSD = \sum_{\mathbf{x}\in\mathbf{T}} \left[I\big(W(\mathbf{x},\mathbf{p})\big) - T(\mathbf{x}) \right]^{\mathbf{2}}$$

Assumptions

- **No errors in the template image boundaries:** only the appearance of the object to be tracked appears in the template image

- **No occlusion**: the entire template is visible in the input image

- **Brightness constancy,**
- **Temporal consistency,**
- **Spatial coherency**

The Lucas-Kanade Tracker

- Uses the Gauss-Newton method for minimization, that is:
  - Applies a first-order approximation of the warp
  - Attempts to minimize the SSD iteratively

B. D. Lucas and T. Kanade (1981), *An iterative image registration technique with an application to stereo vision.* Proceedings of Imaging Understanding Workshop, pages 121-130
S. Baker and I. Matthews, *Lucas-Kanade 20 Years On: A Unifying Framework*, *International Journal of Computer Vision*, 2004

Derivation of the Lucas-Kanade algorithm

$$E = \sum_{\mathbf{x} \in \mathbf{T}} \left[I\left(W(\mathbf{x}, \mathbf{p})\right) - T(\mathbf{x}) \right]^{2}$$

How do I get the initial estimate?

- Assume that an initial estimate of $\mathbf{p}$ is known. Then, we want to find the increment $\Delta\mathbf{p}$ that minimizes

$$\sum_{\mathbf{x} \in \mathbf{T}} \left[I\left(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p})\right) - T(\mathbf{x}) \right]^{2}$$

- First-order Taylor approximation of $I\left(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p})\right)$ yelds to:

$$I\left(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p})\right) \cong I\left(W(\mathbf{x}, \mathbf{p})\right) + \nabla I \frac{\partial W}{\partial \mathbf{p}} \Delta\mathbf{p}$$

$\nabla I = \left[I_x, I_y\right]$ = Image gradient evaluated at $W(\mathbf{x}, \mathbf{p})$

Jacobian of the warp $W(\mathbf{x}, \mathbf{p})$

Derivation of the Lucas-Kanade algorithm

$$E = \sum_{\mathbf{x} \in \mathbf{T}} \left[I\big(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p})\big) - T(\mathbf{x}) \right]^2$$

- By replacing $I\big(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p})\big)$ with its 1st order approximation, we get

$$E = \sum_{\mathbf{x} \in \mathbf{T}} \left[I\big(W(\mathbf{x}, \mathbf{p})\big) + \nabla I \frac{\partial W}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x}) \right]^2$$

- How do we minimize it?
- We differentiate E with respect to $\Delta\mathbf{p}$ and we equate it to zero, i.e.,

$$\frac{\partial E}{\partial \Delta\mathbf{p}} = 0$$

Derivation of the Lucas-Kanade algorithm

$$E = \sum_{\mathbf{x} \in \mathbf{T}} \left[I\big(W(\mathbf{x}, \mathbf{p})\big) + \nabla I \frac{\partial W}{\partial \mathbf{p}} \Delta \mathbf{p} - T(\mathbf{x}) \right]^{\mathbf{2}}$$

$$\frac{\partial E}{\partial \Delta \mathbf{p}} = 2 \sum_{\mathbf{x} \in \mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}} \right]^{\mathrm{T}} \left[I\big(W(\mathbf{x}, \mathbf{p})\big) + \nabla I \frac{\partial W}{\partial \mathbf{p}} \Delta \mathbf{p} - T(\mathbf{x}) \right]$$

$$\frac{\partial E}{\partial \Delta \mathbf{p}} = 0$$

$$2 \sum_{\mathbf{x} \in \mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}} \right]^{\mathrm{T}} \left[I\big(W(\mathbf{x}, \mathbf{p})\big) + \nabla I \frac{\partial W}{\partial \mathbf{p}} \Delta \mathbf{p} - T(\mathbf{x}) \right] = 0 \;\Rightarrow$$

Derivation of the Lucas-Kanade algorithm

Notice that these are NOT matrix products but **pixel-wise** products!

$$\Rightarrow \Delta \mathbf{p} = H^{-1} \sum_{\mathbf{x} \in \mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}} \right]^{\mathrm{T}} \left[T(\mathbf{x}) - I\big(W(\mathbf{x}, \mathbf{p})\big) \right] =$$

$$H = \sum_{\mathbf{x} \in \mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}} \right]^{\mathrm{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}} \right]$$

Second moment matrix (Hessian) of the warped image

What does H look like when the warp is a pure translation?

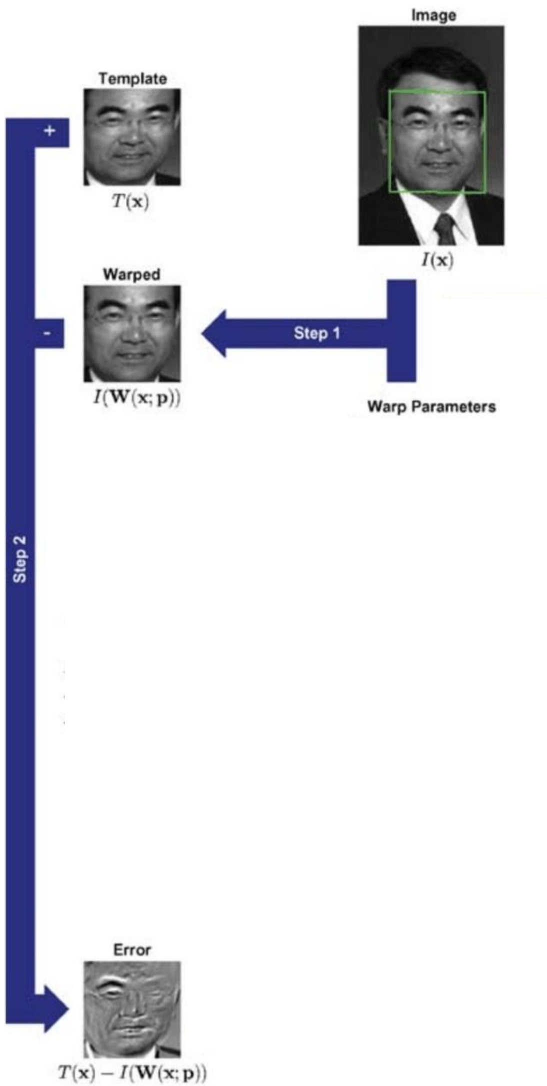
Lucas-Kanade algorithm

$$\Rightarrow \Delta\mathbf{p} = H^{-1} \sum_{\mathbf{x}\in\mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}}\right]^{\mathrm{T}} \left[T(\mathbf{x}) - I\big(W(\mathbf{x},\mathbf{p})\big)\right]$$

1. Warp $I(\mathbf{x})$ with $W(\mathbf{x},\mathbf{p}) \rightarrow I\big(W(\mathbf{x},\mathbf{p})\big)$
2. Compute the error: subtract $I\big(W(\mathbf{x},\mathbf{p})\big)$ from $T(\mathbf{x})$
3. Compute **warped** gradients: $\nabla I = \left[I_x, I_y\right]$, evaluated at $W(\mathbf{x},\mathbf{p})$
4. Evaluate the Jacobian of the warping: $\frac{\partial W}{\partial \mathbf{p}}$
5. Compute steepest descent: $\nabla I \frac{\partial W}{\partial \mathbf{p}}$
6. Compute Inverse Hessian: $H^{-1} = \left[\sum_{\mathbf{x}\in\mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}}\right]^{\mathrm{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}}\right]\right]^{-1}$
7. Multiply steepest descend with error: $\sum_{\mathbf{x}\in\mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}}\right]^{\mathrm{T}} \left[T(\mathbf{x}) - I\big(W(\mathbf{x},\mathbf{p})\big)\right]$
8. Compute $\Delta\mathbf{p}$
9. Update parameters: $\mathbf{p} \leftarrow \mathbf{p} + \Delta\mathbf{p}$
10. Repeat until $\Delta\mathbf{p} < \boldsymbol{\varepsilon}$

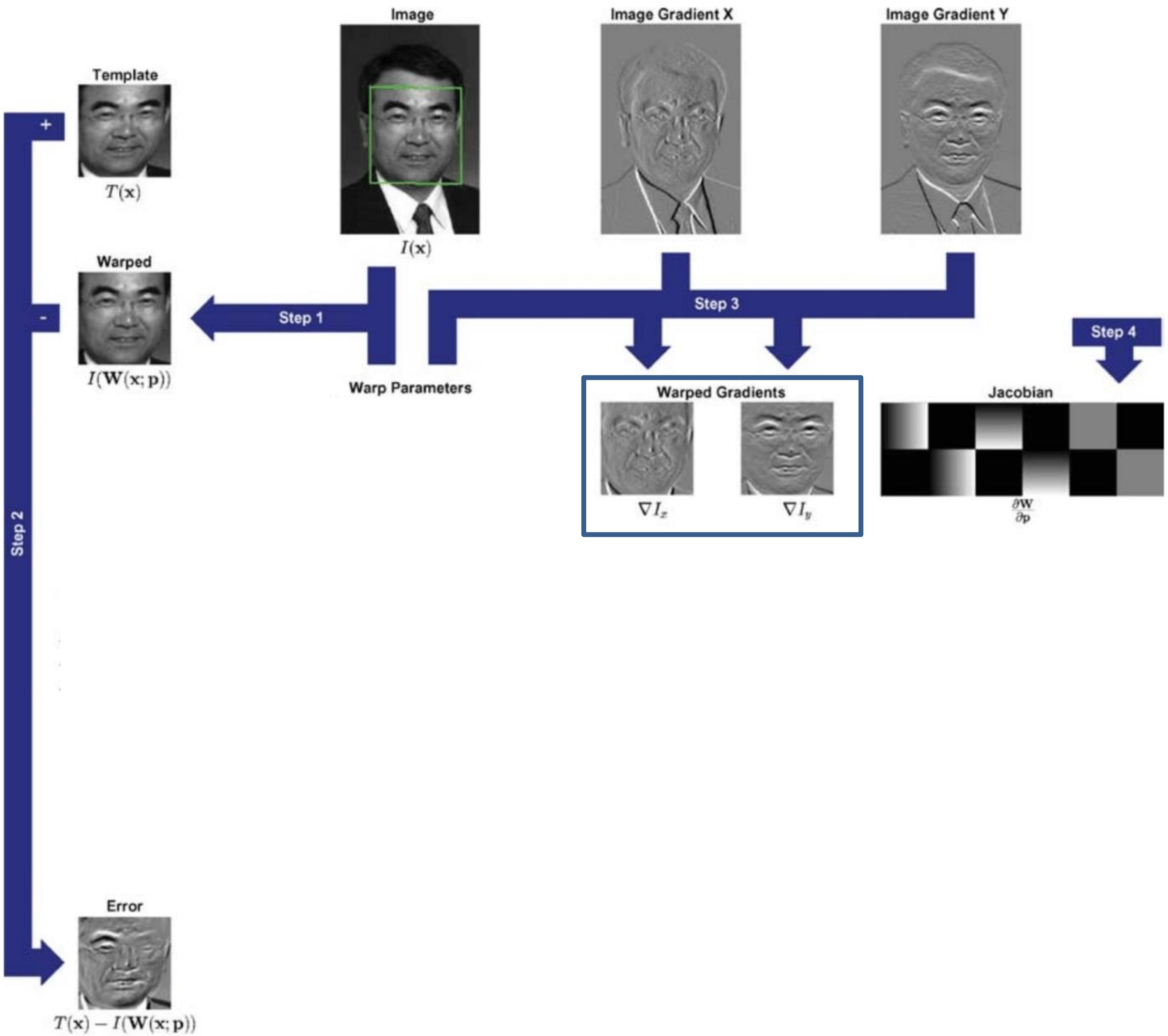
Lucas-Kanade algorithm

$$\Delta \mathbf{p} = H^{-1} \sum_{\mathbf{x} \in \mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}} \right]^{\mathrm{T}} \left[T(\mathbf{x}) - I(W(\mathbf{x}, \mathbf{p})) \right]$$



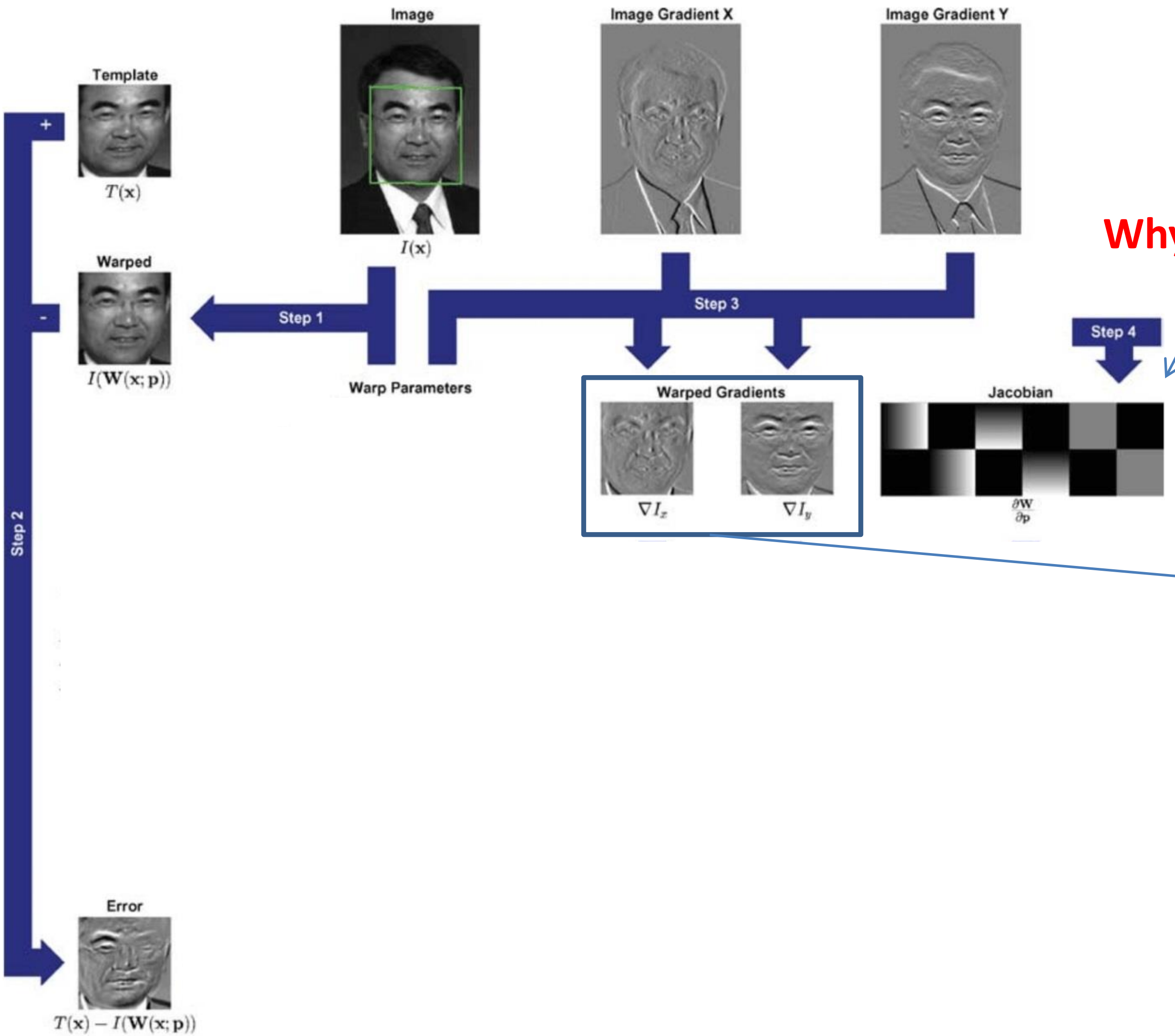
Lucas-Kanade algorithm

$$\Delta\mathbf{p} = H^{-1}\sum_{\mathbf{x}\in\mathbf{T}}\left[\nabla I\frac{\partial W}{\partial\mathbf{p}}\right]^{\mathrm{T}}\left[T(\mathbf{x}) - I\big(W(\mathbf{x},\mathbf{p})\big)\right]$$



Lucas-Kanade algorithm

$$\Delta \mathbf{p} = H^{-1} \sum_{\mathbf{x} \in \mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}} \right]^{\mathrm{T}} \left[T(\mathbf{x}) - I(W(\mathbf{x}, \mathbf{p})) \right]$$



Why does it look like that?

$$\frac{\partial W}{\partial \mathbf{p}} = \begin{bmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{bmatrix}$$

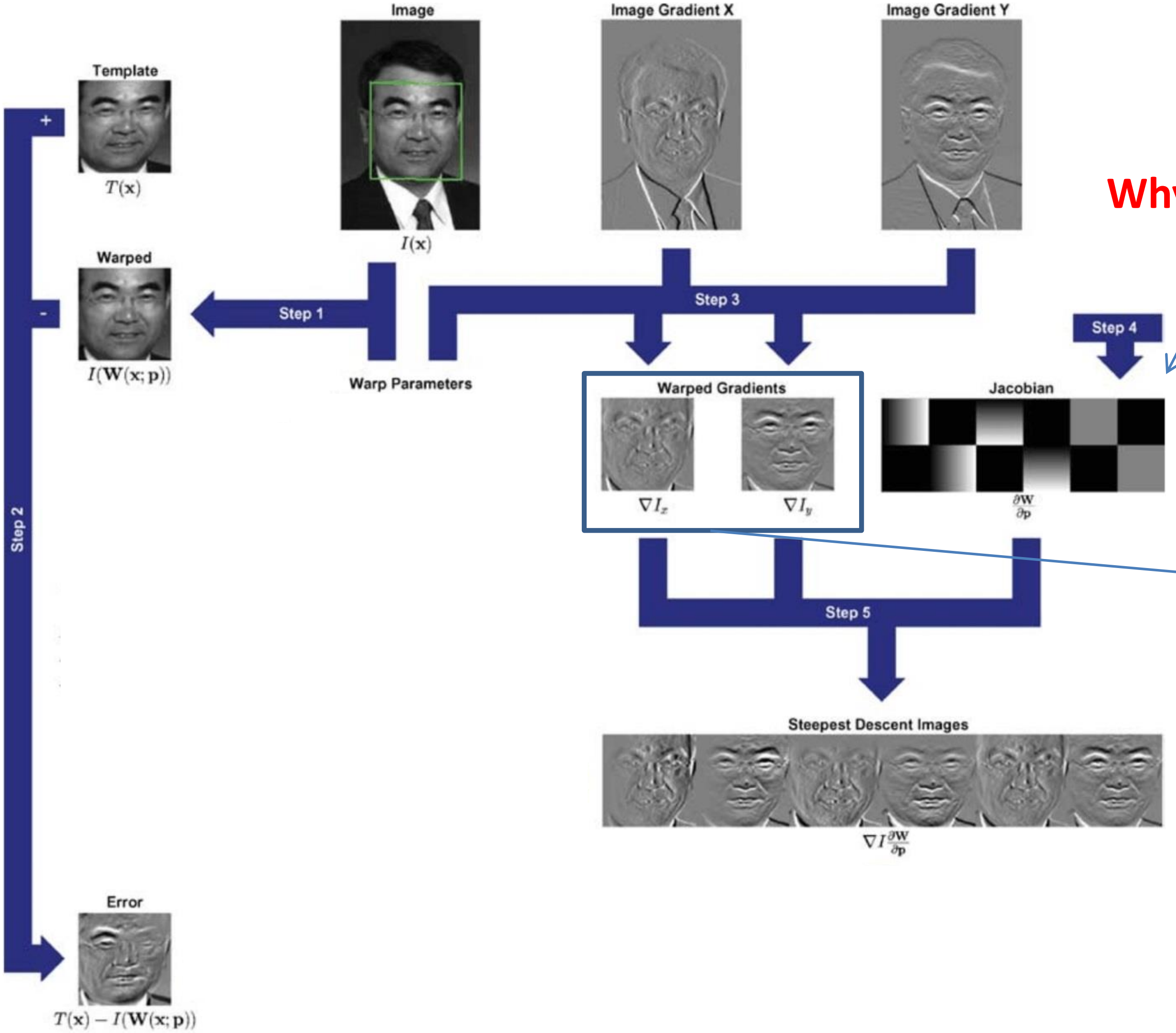
What is the size?

$$2n \times 6n$$

$$n \times 2n$$

Lucas-Kanade algorithm

$$\Delta \mathbf{p} = H^{-1} \sum_{\mathbf{x} \in \mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}} \right]^{\mathrm{T}} \left[T(\mathbf{x}) - I(W(\mathbf{x}, \mathbf{p})) \right]$$



Why does it look like that? (slide 48)

$$\frac{\partial W}{\partial \mathbf{p}} = \begin{bmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{bmatrix}$$

What is the size?

$2n \times 6n$

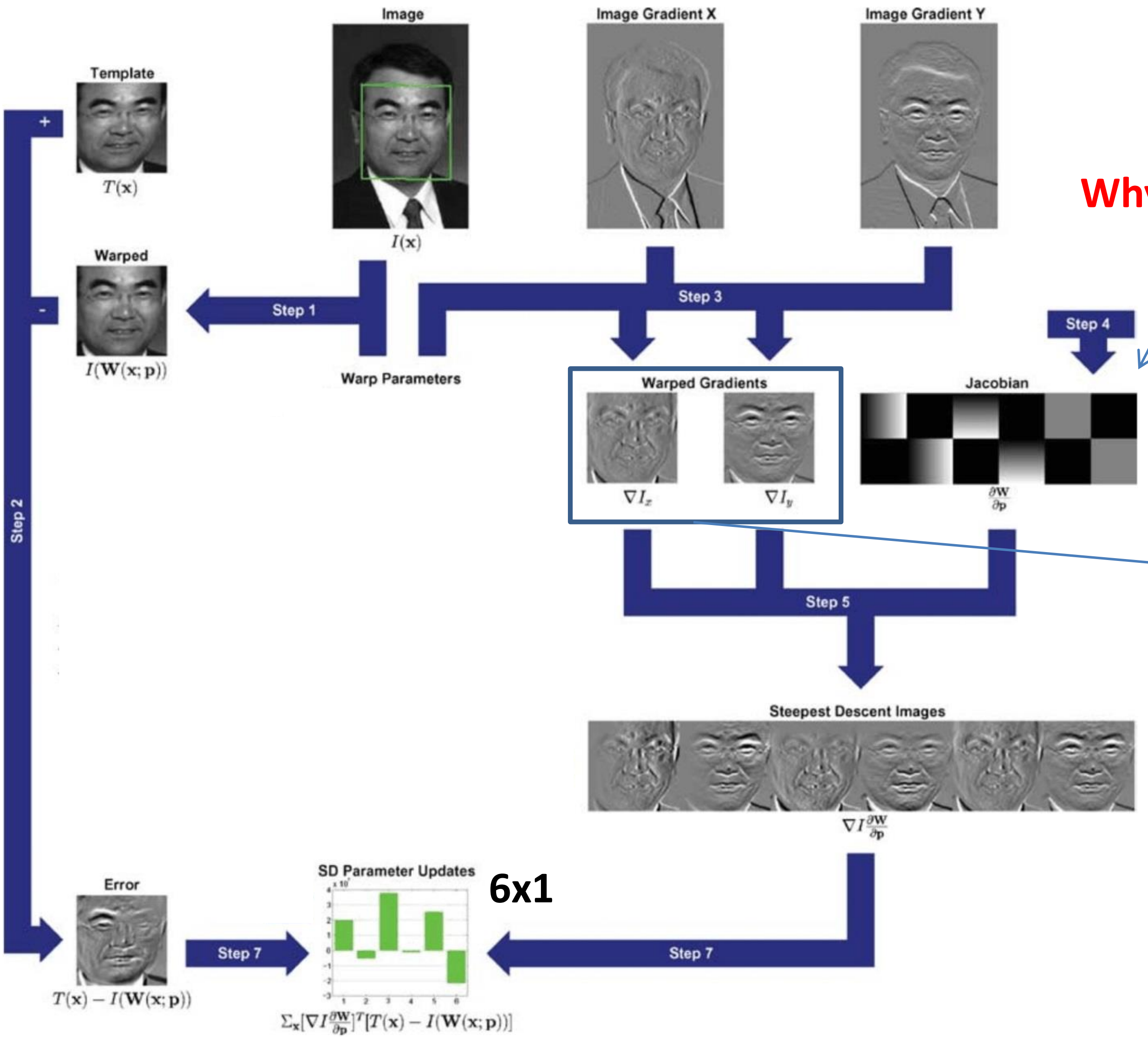
$n \times 2n$

What's its size?

$n \times 6n$

Lucas-Kanade algorithm

$$\Delta \mathbf{p} = H^{-1} \sum_{\mathbf{x} \in \mathbf{T}} \left[\nabla I \frac{\partial W}{\partial \mathbf{p}} \right]^{\mathrm{T}} \left[T(\mathbf{x}) - I(W(\mathbf{x}, \mathbf{p})) \right]$$



6x1

Why does it look like that? (slide 48)

$$\frac{\partial W}{\partial \mathbf{p}} = \begin{bmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{bmatrix}$$

What is the size?

$2n \times 6n$

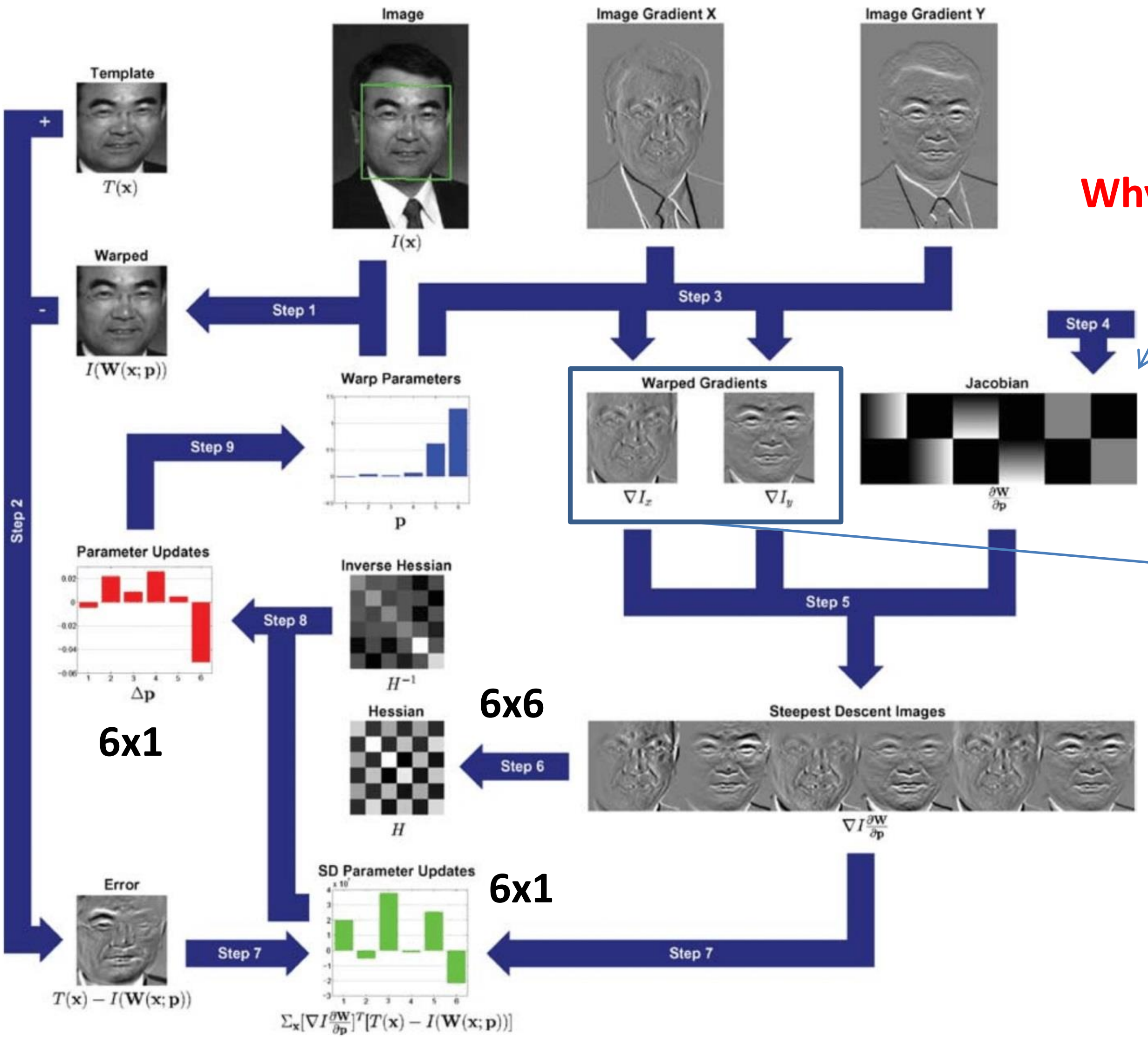
$n \times 2n$

What's its size?

$n \times 6n$

Lucas-Kanade algorithm

$$\Delta\mathbf{p} = H^{-1}\sum_{\mathbf{x}\in\mathbf{T}}\left[\nabla I\frac{\partial W}{\partial \mathbf{p}}\right]^{\mathrm{T}}\left[T(\mathbf{x}) - I(W(\mathbf{x},\mathbf{p}))\right]$$



6x1

6x6

6x1

Why does it look like that? (slide 48)

$$\frac{\partial W}{\partial \mathbf{p}} = \begin{bmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{bmatrix}$$

What is the size?

$2n \times 6n$

$n \times 2n$

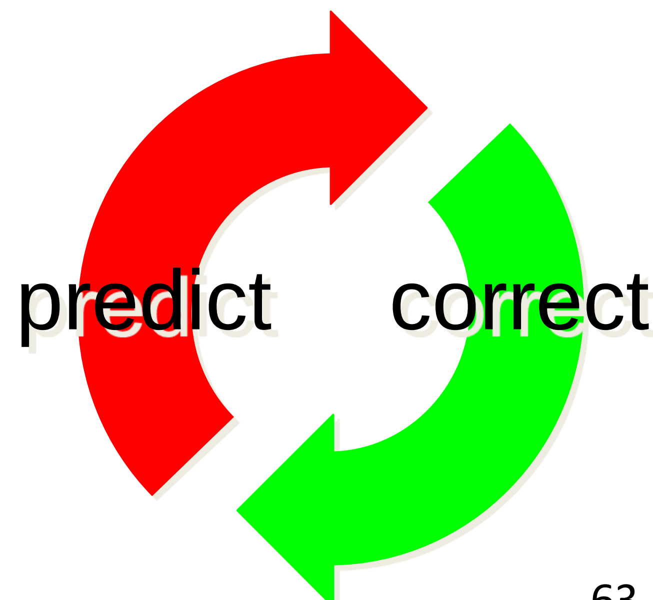
What's its size?

$n \times 6n$

Lucas-Kanade algorithm: Discussion

Lucas-Kanade follows a *predict-correct* cycle

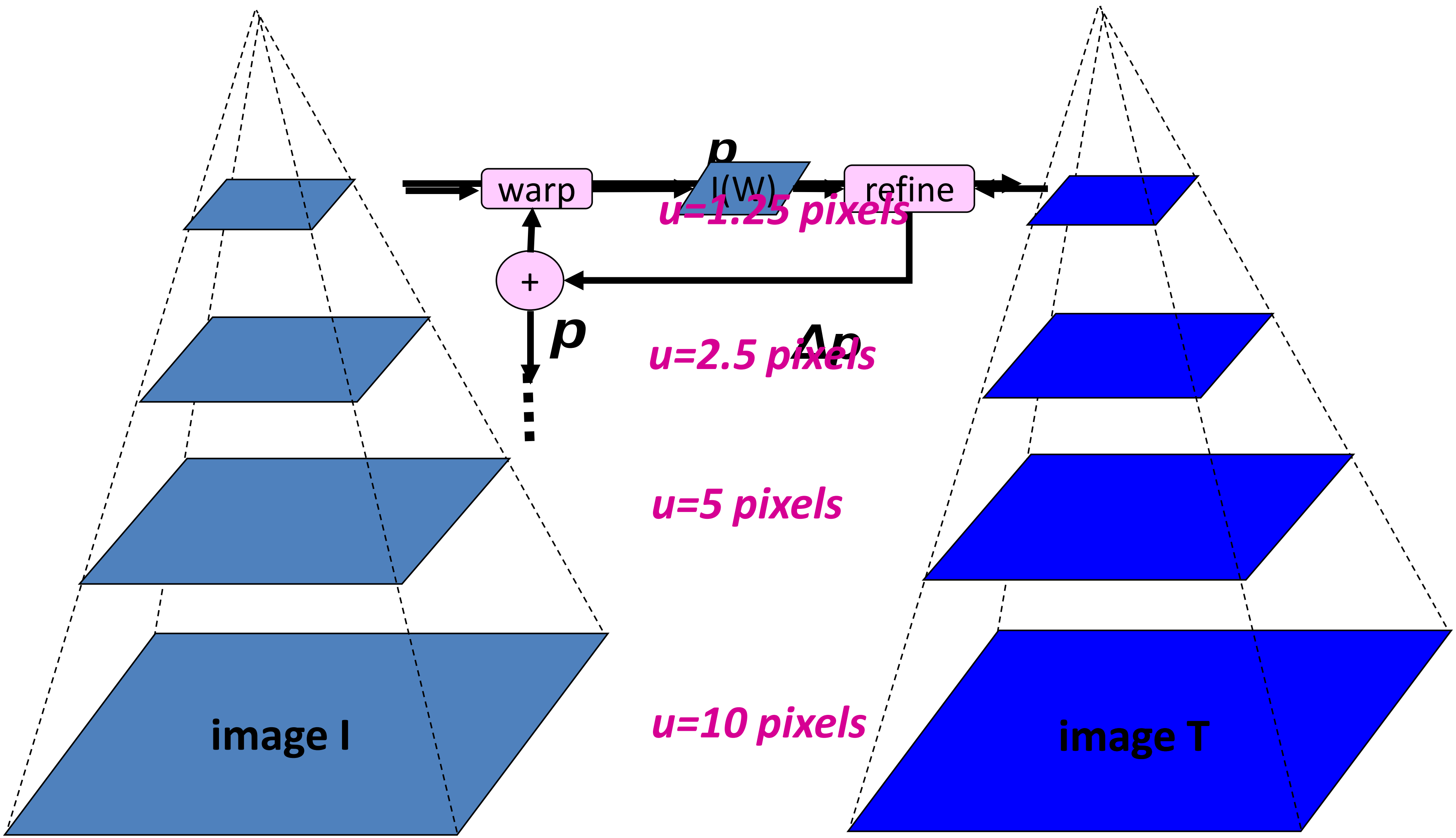
- A *prediction* $I(W(\mathbf{x}, \mathbf{p}))$ of the warped image is computed from an initial estimate
- The *correction* parameter $\Delta\mathbf{p}$ is computed as a function of the error $T(\mathbf{x}) - I(W(\mathbf{x}, \mathbf{p}))$ between the prediction and the template
- The larger this error, the larger the correction applied



Lucas-Kanade algorithm: Discussion

- How to get the initial estimate **p**?
  - E.g., start at rest or use SIFT to find the object and then track it
- When does the Lucas-Kanade fail?
  - If the initial estimate is too far, then the linear approximation does not longer hold -> solution?
    - Pyramidal implementations (see next slide)
- Other problems:
  - Deviations from the mathematical model: object deformations, illumination changes, etc.
  - Occlusions
  - Due to these reasons, tracking may drift -> solution?
    - Update the template with the last image

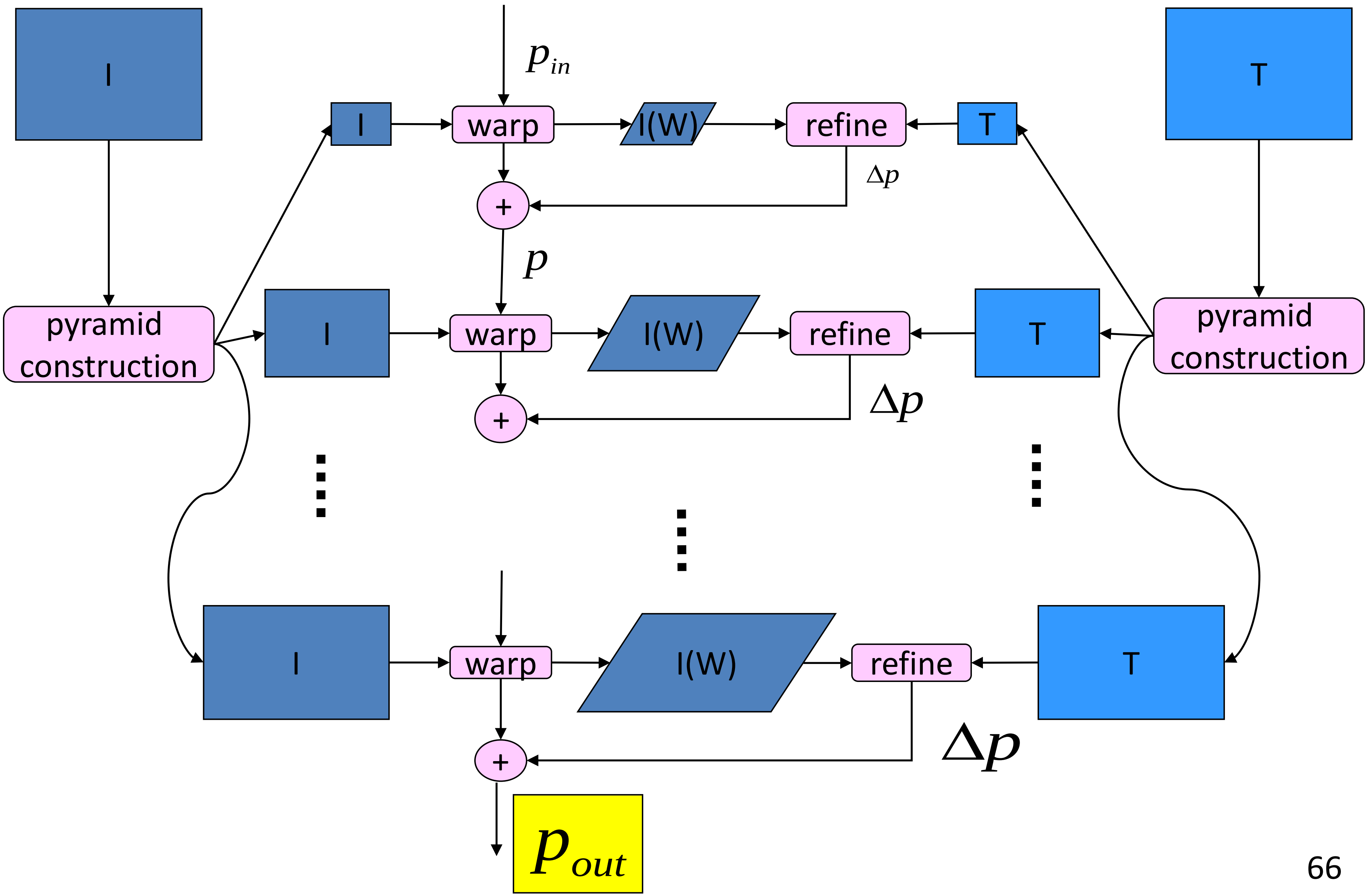
Coarse-to-fine estimation



Pyramid of image I

Pyramid of image T

Coarse-to-fine estimation

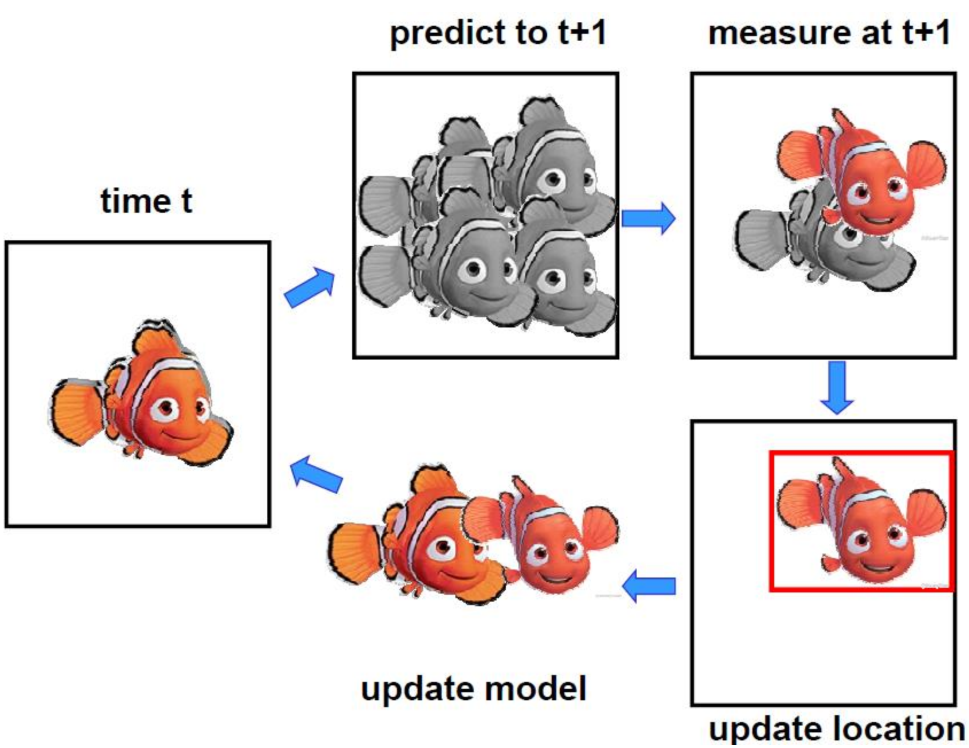


Generalization of Lucas-Kanade

- The same concept (predict/correct) can be applied to tracking of 3D object (in this case, what is the transformation to etimate? What is the template?)

Generalization of Lucas-Kanade

- The same concept (predict/correct) can be applied to tracking of 3D object (in this case, what is the transformation to etimate? What is the template?)
- In order to deal with wrong prediction, it can be implemented in a **Particle-Filter** fashion (using multiple hipotheses that need to be validated)



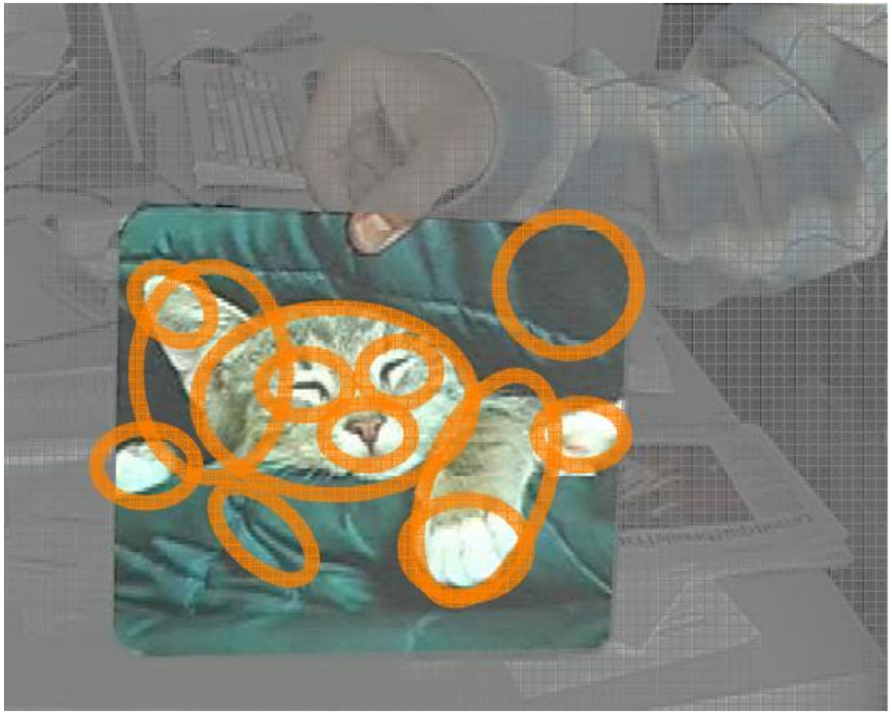
Outline

- Point tracking
- Template tracking
- Tracking by detection of local image features

Tracking by detection of local image features

Step 1: Keypoint detection and matching

- invariant to scale, rotation, or perspective



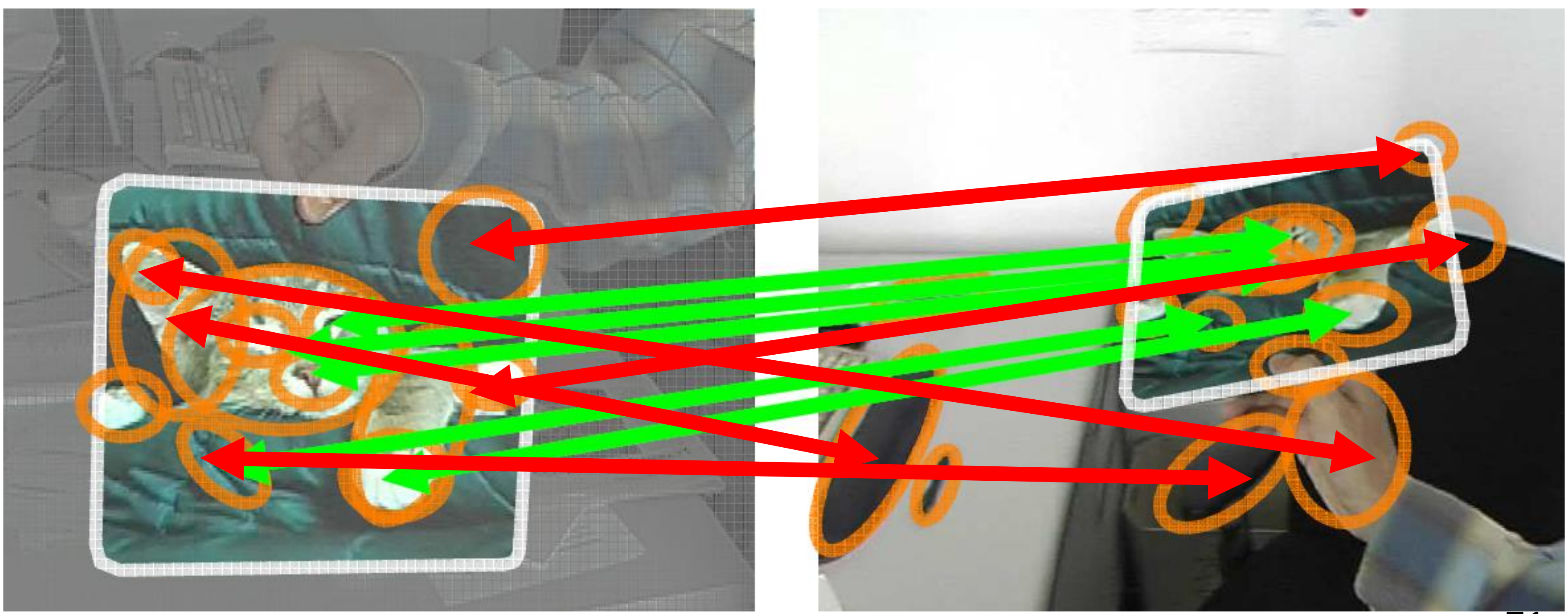
Template image with the object to detect

Current test image

Tracking by detection of local image features

Step 1: Keypoint detection and matching

- invariant to scale, rotation, or perspective



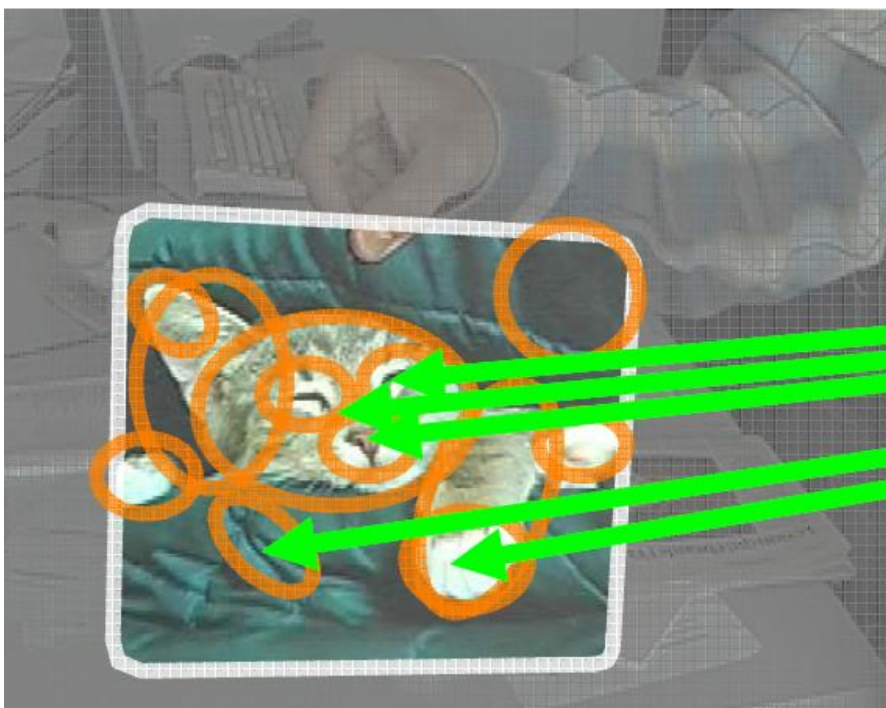
Template image with the object to detect　　　　Current test image

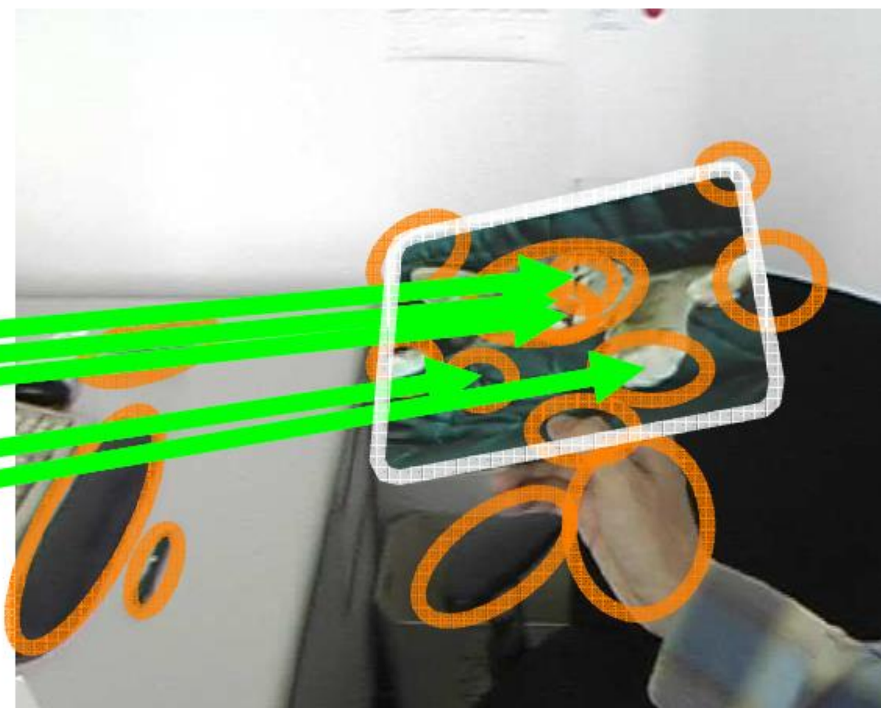
Tracking by detection of local image features

Step 1: Keypoint detection and matching

- invariant to scale, rotation, or perspective

Step 2: Geometric verification (RANSAC)





Template image with the object to detect

Current test image

Tracking by detection of local image features

Tracking issues

- How to segment the object to track from background?
- How to initialize the warping?
- How to handle occlusions
- How to handle illumination changes and non modeled effects?

References

- Chapter 8 of Szeliski's book

Understanding Check

Are you able to answer the following questions?

- Are you able to illustrate tracking with block matching?
- Are you able to explain the underlying assumptions behind differential methods, derive their mathematical expression and the meaning of the M matrix?
- When is this matrix invertible and when not?
- What is the aperture problem and how can we overcome it?
- What is optical flow?
- Can you list pros and cons of block-based vs. differential methods for tracking?
- Are you able to describe the working principle of KLT?
- Are you able to derive the main mathematical expression for KLT?
- What is the Hessian matrix and for which warping function does it coincide to that used for point tracking?
- Can you list Lukas-Kanade failure cases and how to overcome them?
- How does one get the initial guess?
- Can you illustrate the coarse-to-fine Lucas-Kanade implementation?
- Can you illustrate alternative tracking procedures using point features?