



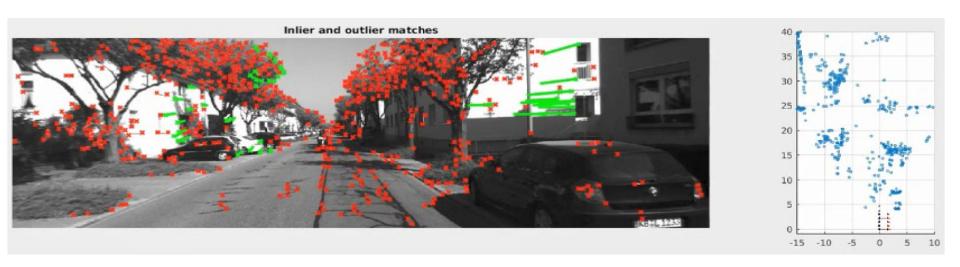
Lecture 09 Multiple View Geometry 3

Davide Scaramuzza

http://rpg.ifi.uzh.ch/

Lab Exercise 7 - Today

- > Room ETH HG E 1.1 from 13:15 to 15:00
- ➤ Work description: P3P algorithm and RANSAC



Outline

- Bundle Adjustment
- SFM with *n* views

Bundle Adjustment (BA) – More in depth in Exercise 9

- Non-linear, simultaneous refinement of structure P^i and motion C = R, T
- It is used after linear estimation of R and T (e.g., after 8-point algorithm)
- Computes C, P^i by minimizing the Sum of Squared Reprojection Errors:

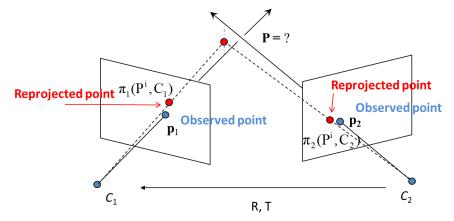
$$(P^{i}, C_{2}) = \arg\min_{P^{i}, C_{1}, C_{2}} \sum_{i=1}^{N} \|p_{1}^{i} - \pi_{1}(P^{i}, C_{1})\|^{2} + \|p_{2}^{i} - \pi_{2}(P^{i}, C_{2})\|^{2}$$

NB: here, by C_1 , C_2 we denote the **pose of** each camera in the **world** frame

 Can be minimized using Levenberg-Marquardt (more robust than Gauss-Newton to local minima)

In order to not get stuck in local minima, the initialization should be close the

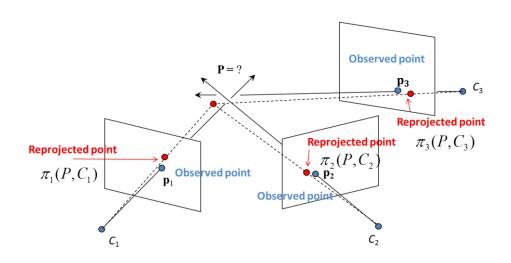
minimum

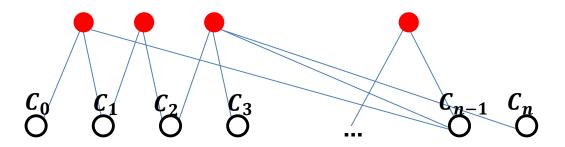


Bundle Adjustment (BA) for n Views

Minimizes the Sum of Squared Reprojection Errors over each view k

$$(P^{i}, C_{k}) = \arg\min_{P^{i}, C_{k}} \sum_{k} \sum_{i} \|p_{k}^{i} - \pi_{k}(P^{i}, C_{k})\|^{2}$$





Huber and Tukey Norms

To prevent that large reprojection errors can negatively influence the optimization, a more robust norm $\rho()$ is used instead of the L_2 :

$$(P^i, C_k) = \operatorname{arg\,min}_{P^i, C_k} \sum_{k} \sum_{i} \rho(p_k^i - \pi_k(P^i, C_k))$$

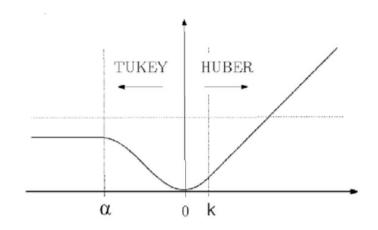
 $\rho()$ is a robust cost function (**Huber or Tukey**) to penalize wrong matches:

> Huber norm:

$$\rho(x) = \begin{cases} x^2 & \text{if } |x| \le k \\ k(2|x| - k) & \text{if } |x| \ge k \end{cases}$$

> Tukey norm:

$$\rho(x) = \begin{cases} \alpha^2 & \text{if } |x| \ge \alpha \\ \alpha^2 \left(1 - \left(1 - \left(\frac{x}{\alpha} \right)^2 \right)^3 \right) & \text{if } |x| \le \alpha \end{cases}$$



These formulas are not asked at the exam but their plots and meaning is asked ©

Outline

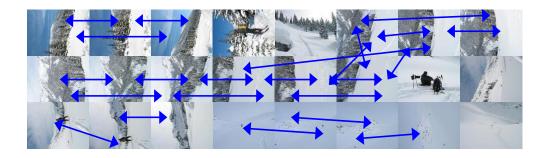
- Bundle Adjustment
- SFM with *n* views

Structure From Motion with n Views

- Compute initial structure and motion
 - Hierarchical SFM
 - Sequential SFM
- Refine simultaneously structure and motion through BA

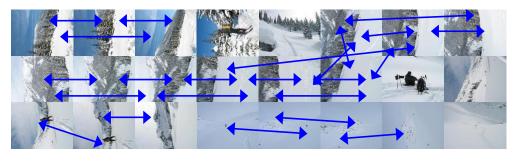
Hierarchical SFM

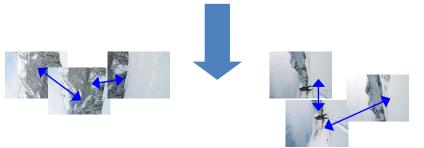
1. Extract and match features between nearby frames



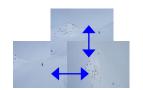
Hierarchical SFM

- 1. Extract and match features between nearby frames
- 2. Identify clusters consisting of 3 nearby frames:
- 3. Compute SFM for 3 views:
 - Compute SFM between
 1 and 2 and build point cloud
 - Then merge 3rd view by running 3-point RANSAC between point cloud and 3rd view









Hierarchical SFM

- 1. Extract and match features between nearby frames
- 2. Identify clusters consisting of 3 nearby frames:
- 3. Compute SFM for 3 views:
 - Compute SFM between
 1 and 2 and build point cloud
 - Then merge 3rd view by running 3-point RANSAC between point cloud and 3rd view
- 4. Merge clusters pairwise and refine (BA) both structure and motion

How do you merge clusters?

Hierarchical SFM: Example

- > Reconstruction from 150,000 images from Flickr associated with the tags "Rome"
- > 4m 3D points. Cloud of 496 computers. 21 hours of computation!
- Paper: "Building Rome in a Day", ICCV'09: http://grail.cs.washington.edu/rome/
 University of Washington, 2009 Most influential paper of 2009 (link)



Structure From Motion with n Views

- Compute initial structure and motion
 - Hierarchical SFM
 - Sequential SFM
- Refine simultaneously structure and motion through BA

Sequential SFM - also called Visual Odometry (VO)

- Initialize structure and motion from 2 views (bootstrapping)
- For each additional view
 - Determine pose (localization)
 - > Extend structure, i.e., extract and triangulate new features (mapping)
 - > Refine structure and motion through Bundle Adjustment (BA) (optimization)

A Brief history of VO

➤ **1980**: First known VO real-time implementation on a robot by Hans Moraveck PhD thesis (Stanforfd/NASA/JPL) for Mars rovers using one sliding camera (sliding stereo).



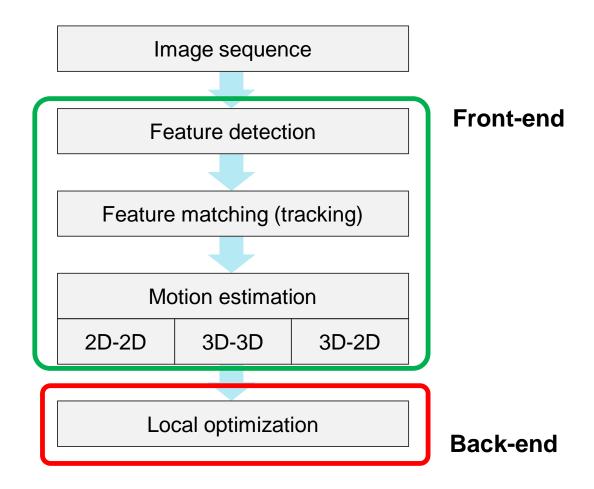
A Brief history of VO

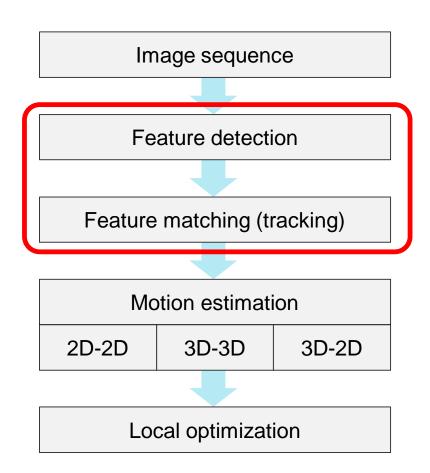
- ➤ 1980: First known VO real-time implementation on a robot by Hans Moraveck PhD thesis (Stanforfd/NASA/JPL) for Mars rovers using one sliding camera (sliding stereo).
- 1980 to 2000: The VO research was dominated by NASA/JPL in preparation of the 2004 mission to Mars
- > 2004: VO was used on a robot on another planet: Mars rovers Spirit and Opportunity (see seminal paper from NASA/JPL, 2007)
- 2004. VO was revived in the academic environment by David Nister's «Visual Odometry» paper. The term VO became popular.



More about VO history and tutorials

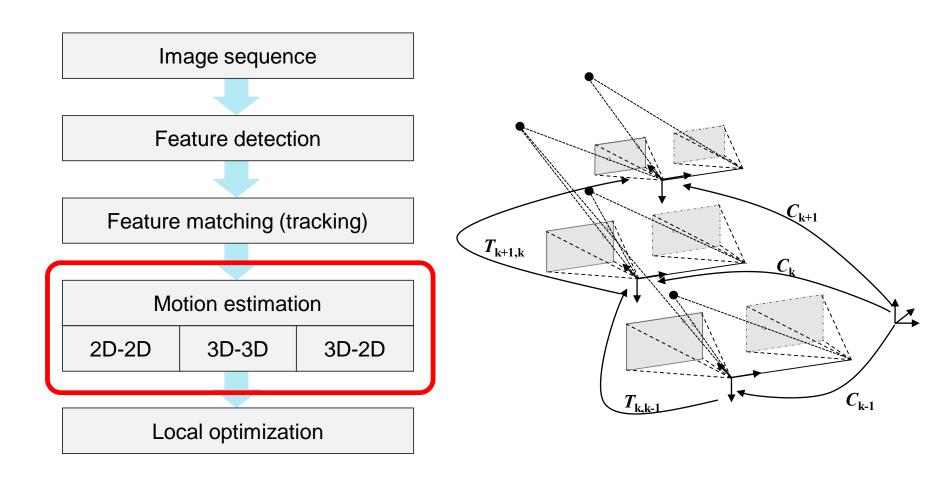
- ➤ Scaramuzza, D., Fraundorfer, F., Visual Odometry: Part I The First 30 Years and Fundamentals, IEEE Robotics and Automation Magazine, Volume 18, issue 4, 2011. PDF
- Fraundorfer, F., Scaramuzza, D., Visual Odometry: Part II Matching, Robustness, and Applications, *IEEE Robotics and Automation Magazine*, Volume 19, issue 1, 2012. PDF
- ➤ C. Cadena, L. Carlone, H. Carrillo, Y. Latif, D. Scaramuzza, J. Neira, I.D. Reid, J.J. Leonard, Past, Present, and Future of Simultaneous Localization and Mapping: Toward the Robust-Perception Age, IEEE Transactions on Robotics, Vol. 32, Issue 6, 2016. PDF

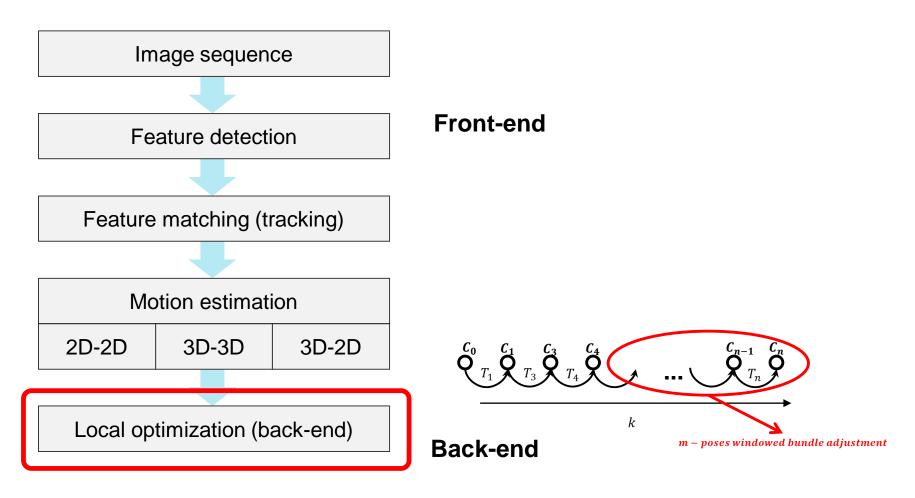






Example features tracks



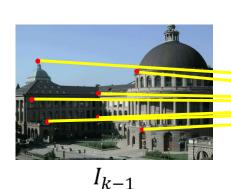


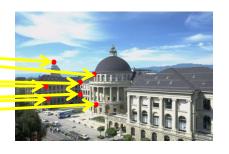
Motion from Image Feature Correspondences

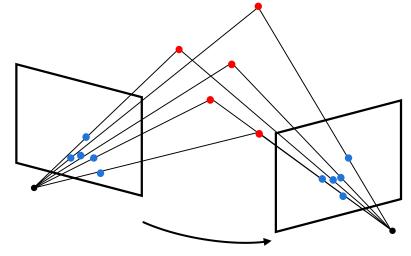
- \triangleright Both feature points f_{k-1} and f_k are specified in 2D
- > The minimal-case solution involves **5-point** correspondences
- > The solution is found by minimizing the reprojection error:

$$T_k = \begin{bmatrix} R_{k,k-1} & t_{k,k-1} \\ 0 & 1 \end{bmatrix} = \arg\min_{T_k} \sum_i ||p_k^i - \hat{p}_{k-1}^i||^2$$

Popular algorithms: 5- and 8-point algorithms [Hartley'97, Nister'06]







3D-to-2D (already seen: Lecture 03)

Motion from 3D Structure and Image Correspondences

- $\succ f_{k-1}$ is specified in **3D** and f_k in **2D**
- \triangleright This problem is known as *camera resection* or PnP (Perspective from *n* Points)
- ➤ The minimal-case solution involves **3 correspondences** (**+1** for disambiguating the 4 solutions)
- > The solution is found by minimizing the reprojection error:

$$T_k = \begin{bmatrix} R_{k,k-1} & t_{k,k-1} \\ 0 & 1 \end{bmatrix} = \arg\min_{X^i, C_k} \sum_{i,k} \|p_k^i - g(X^i, C_k)\|^2$$

Popular algorithms: P3P [Gao'03, Kneip'11] x_1 y_1 x_1 y_1 x_2

Davide Scaramuzza – University of Zurich – Ro

3D-to-3D

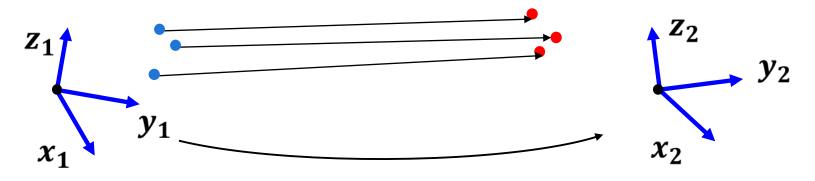
Motion estimation			
2D-2D	3D-2D	3D-3D	

Motion from 3D-3D Point Correspondences (point cloud registration)

- \triangleright Both f_{k-1} and f_k are specified **in 3D**. To do this, it is necessary to triangulate 3D points (e.g. use a stereo camera)
- > The minimal-case solution involves 3 non-collinear correspondences
- ➤ The solution is found by minimizing the 3D-3D Euclidean distance:

$$T_k = \begin{bmatrix} R_{k,k-1} & t_{k,k-1} \\ 0 & 1 \end{bmatrix} = \arg\min_{T_k} \sum_i ||\tilde{X}_k^i - T_k \tilde{X}_{k-1}^i||$$

Popular algorithm: [Arun'87] for global registration plus local refinement with Bundle Adjustment (BA)



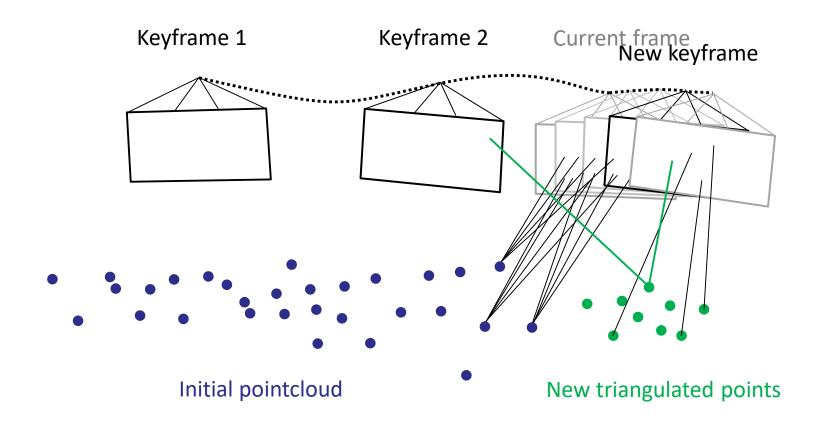
Arun, Huang, Blostein, "Least-squares fitting of two 3-d point sets," PAMI'87. PDF

Motion Estimation: Summary

Type of correspondences	Monocular	Stereo
2D-2D	X	
3D-2D	X	X
3D-3D		X

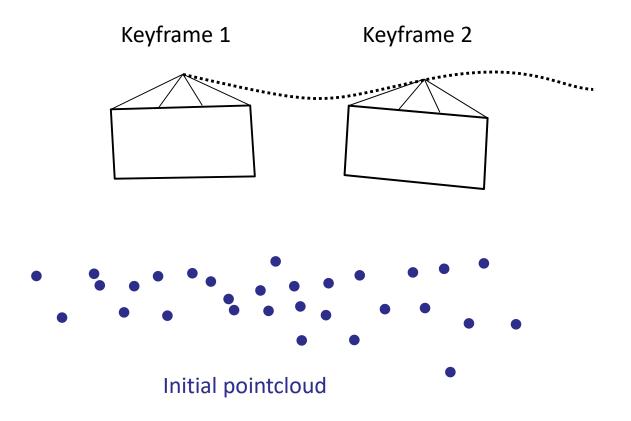
Case Study: Monocular Visual Odometry

Case study: Monocular VO



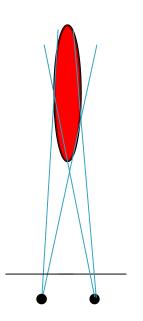
This pipeline was initially proposed in PTAM (Parallel Tracking & Mapping) [Klein, ISMAR'07]

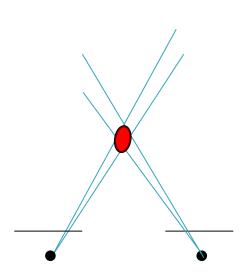
- Bootstrapping (i.e., initialization)
 - ➤ Initialize structure and motion from 2 views: e.g., 5- or 8-point RANSAC
 - Refine structure and motion (Bundle Adjustment)
 - ➤ How far should the two frames (i.e., keyframes) be?



Skipping frames (Keyframe Selection)

When frames are taken at nearby positions compared to the scene distance, 3D points will exibit large uncertainty



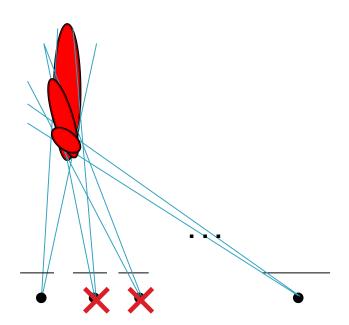


Small baseline → large depth uncertainty

Large baseline → small depth uncertainty

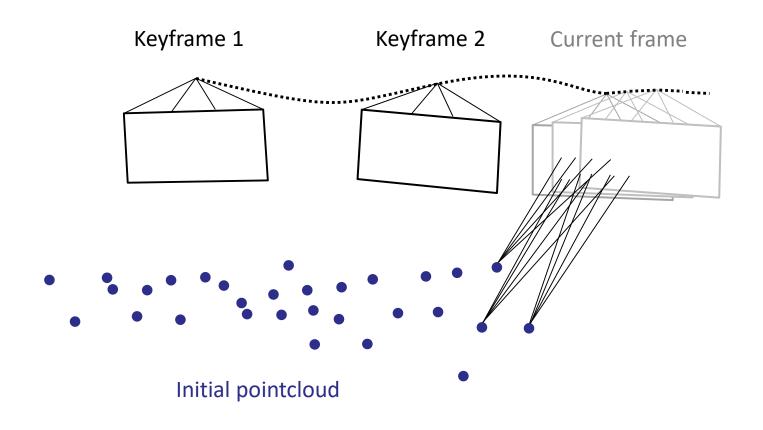
Skipping frames (Keyframe Selection)

- When frames are taken at nearby positions compared to the scene distance, 3D points will exibit large uncertainty
- One way to avoid this consists of skipping frames until the average uncertainty of the 3D points decreases below a certain threshold. The selected frames are called keyframes
- > Rule of the thumb: add a keyframe when $\frac{keyframe\ distance}{average-depth}$ > threshold (~10-20 %)



Localization

- Given a 3D point cloud (map), determine the pose of each additional view
 - ➤ How?
 - How long can I do that?



Localization

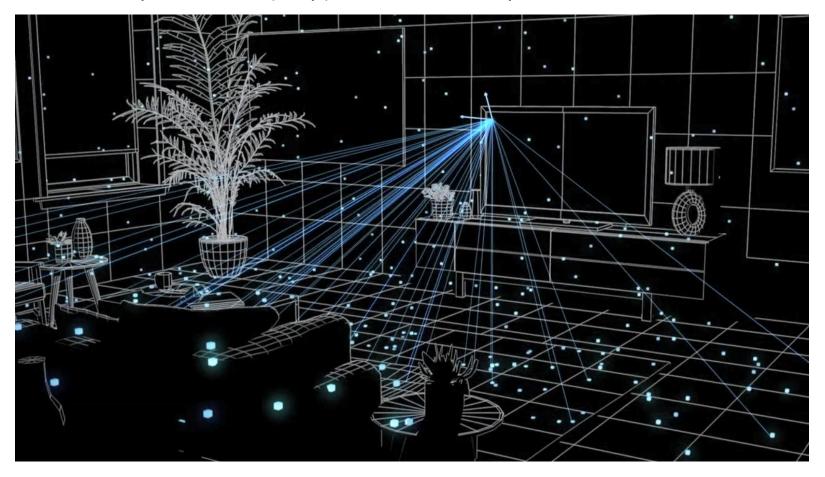
- Given a 3D point cloud (map), determine the pose of each additional view
 - ➤ How?
 - How long can I do that?

Recall:

- PnP problem (Perspective from n Points)
- What's the minimal number of required point correspondences?
 - ➤ Lecture 3:
 - ➤ 6 for DLT algorithm (linear solution)
 - ➤3 (+1) for P3P algorithm (non-linear solution)

Localization

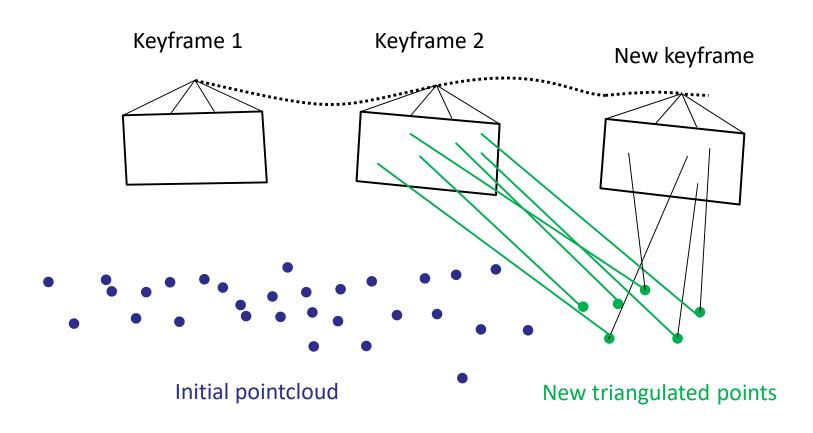
Given a 3D point cloud (map), determine the pose of each additional view



<u>Video</u> of Oculus Insight (the VIO used in Oculus Quest): built by former <u>Zurich-Eye team</u>, today Oculus Zurich. Dr. Christian Forster (Oculus Zurich & co-founder of Zurich-Eye) will give a lecture on Nov. 28 34

Extend Structure (i.e., mapping)

- > Extract and triangulate new features
 - Is it necessary to do this for every frame or can we just do it for keyframes?
 - What are the pros and cons?



Monocular Visual Odometry: putting all pieces together

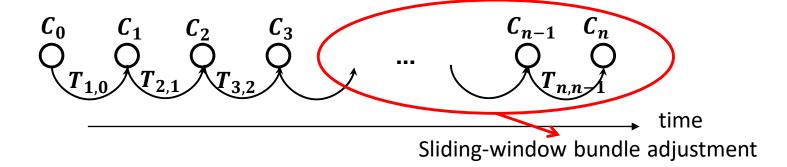
• Let the relative motion T_k from images I_{k-1} to image I_k

$$T_{k,k-1} = \begin{bmatrix} R_{k,k-1} & t_{k,k-1} \\ 0 & 1 \end{bmatrix}$$

Concatenate adjacent transformations to recover the current pose:

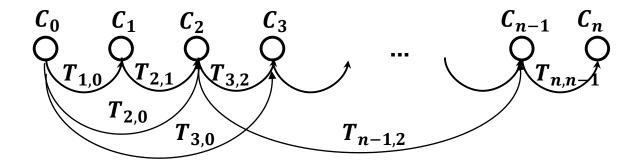
$$C_n = C_{n-1} T_{n,n-1}$$

 Optimize over the last m poses to refine the trajectory (Pose-Graph or Bundle Adjustment)



Pose-Graph Optimization

So far we assumed that the transformations are between consecutive frames

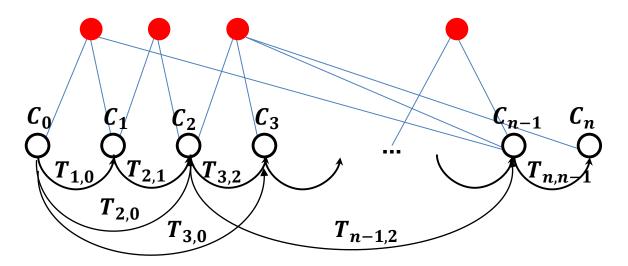


 \succ Transformations can be computed also between **non-adjacent frames** T_{ij} (e.g., when features from previous keyframes are still observed). They can be used as additional constraints to improve cameras poses by minimizing the following:

$$C_k = argmin_{C_k}, \sum_{i} \sum_{j} ||C_i - C_j T_{ij}||^2$$

- For efficiency, only the last m keyframes are used
- Gauss-Newton or Levenberg-Marquadt are typically used to minimize it. For large graphs, efficient open-source tools: g2o, GTSAM, SLAM++, Google Ceres

Bundle Adjustment (BA)



Similar to pose-graph optimization but it also optimizes 3D points

$$X^{i}, C_{k} = argmin_{X^{i}, C_{k}}, \sum_{i} \sum_{k} \rho \left(p_{k}^{i} - \pi(X^{i}, C_{k}) \right)$$

- $\triangleright \rho_H$ () is a robust cost function (e.g., **Huber or Tukey cost**) to penalize wrong matches
- In order to not get stuck in local minima, the initialization should be close to the minimum
- Gauss-Newton or Levenberg-Marquadt are typically used to minimize it. For large graphs, efficient open-source tools: g2o, GTSAM, SLAM++, Google Ceres

Bundle Adjustment vs Pose-graph Optimization

- > BA is **more precise** than pose-graph optimization because it adds additional constraints (*landmark constraints*)
- > But **more costly**: $O((qM + lN)^3)$ with M and N being the number of points and cameras poses and q and l the number of parameters for points and camera poses. Workarounds:
 - A small window size limits the number of parameters for the optimization and thus makes real-time bundle adjustment possible.
 - It is possible to reduce the computational complexity by just optimizing over the camera parameters and keeping the 3-D landmarks fixed, e.g., (motion-only BA)

Loop Closure Detection (i.e., Place Recognition)

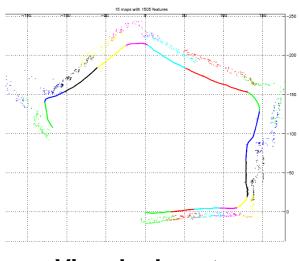
> Relocalization problem:

- During VO, tracking can be lost (due to occlusions, low texture, quick motion, illumination change)
- > Solution: **Re-localize** camera pose and continue
- Loop closing problem
 - When you go back to a previously mapped area:
 - Loop detection: to avoid map duplication
 - Loop correction: to compensate the accumulated drift
 - In both cases you need a place recognition technique

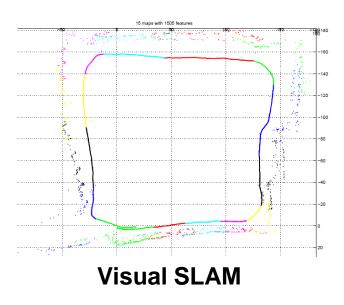
We will address place recognition in Lecture 12

Recall: VO vs. Visual SLAM

Visual SLAM = visual odometry + loop detection + graph optimization



Visual odometry



Open Source Monocular VO and SLAM algorithms

- > PTAM [Klein, 2007] -> Oxford, Murray's lab
- > ORB-SLAM [Mur-Artal, T-RO, 15] -> Zaragoza, Tardos' lab
- > LSD-SLAM [Engel, ECCV'14] -> Munich, Cremers' lab
- > **DSO** [Engel'16] -> Munich, Cremers' lab
- > **SVO** [Forster, ICRA'14, TRO'17] -> Zurich, Scaramuzza's lab

PTAM: Parallel Tracking and Mapping for Small AR Workspaces

Parallel Tracking and Mapping for Small AR Workspaces

ISMAR 2007 video results

Georg Klein and David Murray Active Vision Laboratory University of Oxford

ORB-SLAM [Mur-Artal, TRO'15]

- Feature based
 - FAST corners + ORB descriptors
 - ORB: binary descriptor, very fast to compute and match (Hamming distance)
 - Minimizes reprojection error
- Includes:
 - Loop closing
 - Relocalization
 - Final optimization
- > Real-time (30Hz)

Download from

http://webdiis.unizar.es/~raulmur/orbslam/

ORB-SLAM

Raúl Mur-Artal, J. M. M. Montiel and Juan D. Tardós

{raulmur, josemari, tardos} @unizar.es



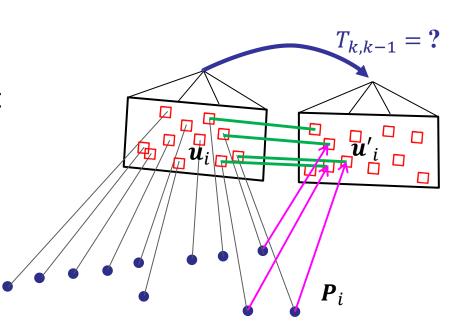


Feature-based methods

- 1. Extract & match features + RANSAC
- 2. Bundle Adjust by minimizing the **Reprojection Error**

$$T_{k,k-1} = \arg\min_{T} \sum_{i} \|\boldsymbol{u'}_{i} - \boldsymbol{u}_{i}\|_{\Sigma}^{2}$$

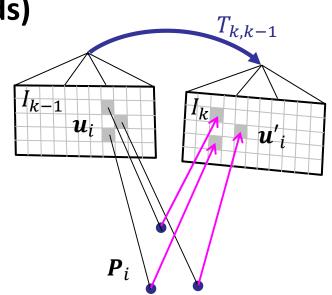
where $\boldsymbol{u'}_{i} = \pi(\boldsymbol{P}_{i}, T_{k,k-1})$



Direct methods (photometric methods)

1. No feature extraction & no RANSAC. Instead, directly minimize **Photometric Error**:

$$\begin{split} T_{k,k-1} &= \arg\min_{T} \sum_{i} \|I_k(\boldsymbol{u'}_i) - I_{k-1}(\boldsymbol{u}_i)\|_{\sigma}^2 \\ \text{where} \quad \boldsymbol{u'}_i &= \pi\big(\boldsymbol{P}_i, T_{k,k-1}\big) \end{split}$$



Feature-based methods

- 1. Extract & match features + RANSAC
- 2. Bundle Adjust by minimizing the **Reprojection Error**

$$T_{k,k-1} = \arg\min_{T} \sum_{i} \|\boldsymbol{u'}_{i} - \boldsymbol{u}_{i}\|_{\Sigma}^{2}$$

where $\boldsymbol{u'}_{i} = \pi(\boldsymbol{P}_{i}, T_{k,k-1})$

- ✓ Large frame-to-frame motions
- ✓ Accuracy: Efficient optimization of structure and motion (Bundle Adjustment)
- Slow due to costly feature extraction and matching
- × Matching Outliers (RANSAC)

Direct methods (photometric me ✓

1. No feature extraction & no RANSAC. Inst directly minimize **Photometric Error**:

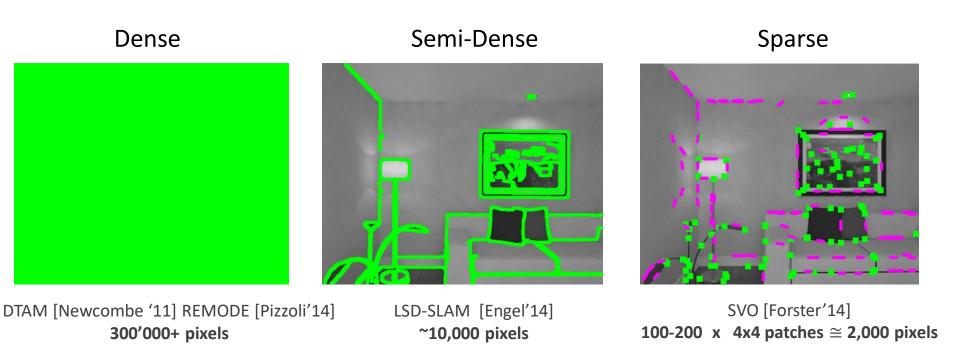
$$T_{k,k-1} = \arg\min_{T} \sum_{i} ||I_{k}(\boldsymbol{u'}_{i}) - I_{k-1}(\boldsymbol{u}_{i})||_{\sigma}^{2}$$

where
$$\mathbf{u'}_i = \pi(\mathbf{P}_i, T_{k,k-1})$$

- All information in the image can be exploited (precision, robustness)
- ✓ Increasing camera frame-rate reduces computational cost per frame
- × Limited frame-to-frame motion
- > Joint optimization of dense structure and motion too expensive

Irani, Anandau, All about direct methods, Springer'99. PDF

Direct Methods: Dense vs Semi-dense vs Sparse [TRO'16]

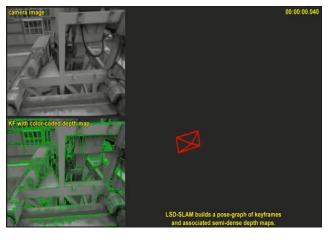


Direct Methods: Dense vs Semi-dense vs Sparse [TRO'16]

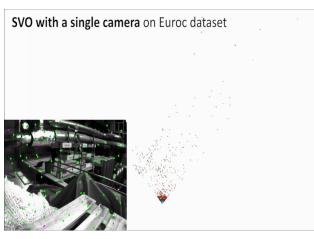
Dense

Live incremental reconstruction of a large scene

Semi-Dense



Sparse



DTAM [Newcombe '11] REMODE [Pizzoli'14] 300'000+ pixels

Texture mapped model

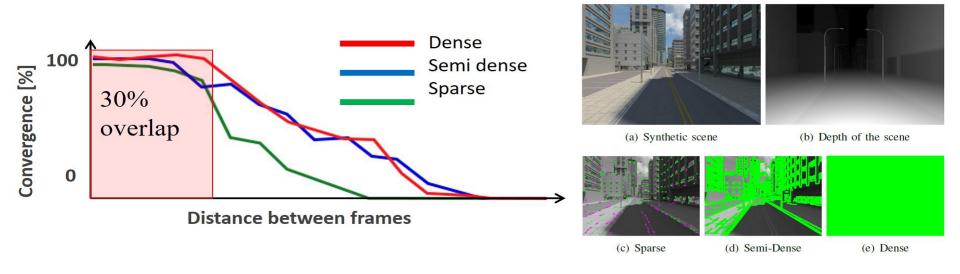
Inverse depth solution

LSD-SLAM [Engel'14] ~10,000 pixels

SVO [Forster'14] DSO [Engel'17] 100-200 x 4x4 patches \cong 2,000 pixels

Direct Methods: Dense vs Semi-dense vs Sparse [TRO'16]

Robustness to motion baseline (computed from 1,000 Blender simulations)



- Dense and Semi-dense behave similarly
 - weak gradients are not informative for the optimization)
- Dense only useful with motion blur and defocus
- > Sparse methods behave equally well for image overlaps up to 30%
- [Forster, et al., SVO: Semi Direct Visual Odometry for Monocular and Multi-Camera Systems, TRO'17]
- Multi-FOV Zurich Urban Dataset: http://rpg.ifi.uzh.ch/fov.html

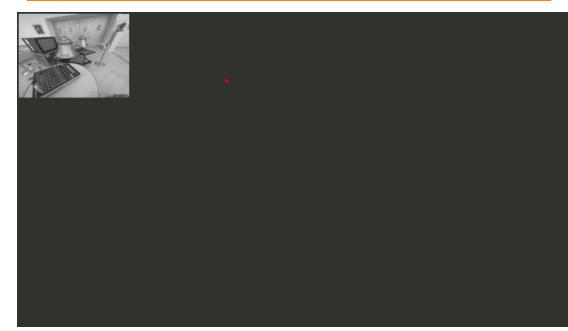
Images from the synthetic Multi-FOV Zurich Urban Dataset

LSD-SLAM [Engel, ECCV'14]

- Direct (photometric error) + Semi-Dense formulation
 - 3D geometry represented as semi-dense depth maps
 - Minimizes photometric error
 - Separateley optimizes poses & structure
- Includes:
 - Loop closing
 - Relocalization
 - Final optimization

> Real-time (30Hz)

Download from https://vision.in.tum.de/research/vslam/lsdslam



DSO [Engel, PAMI'17]

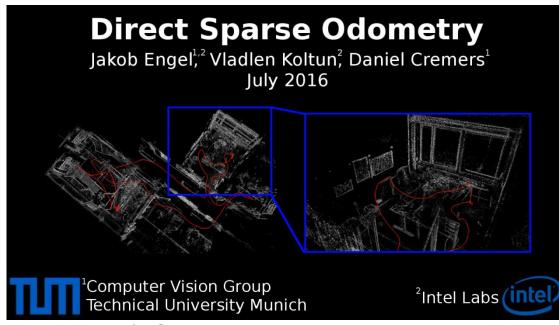
Download from

https://vision.in.tum.de/research/vslam/dso

- > **Direct** (photometric error) + **Sparse** formulation
 - 3D geometry represented as sparse large gradients
 - Minimizes photometric error
 - Jointly optimizes poses & structure (sliding window)
 - Incorporate photometric correction to compensate exposure time change

$$E_{\mathbf{p}j} := \sum_{\mathbf{p} \in \mathcal{N}_{\mathbf{p}}} w_{\mathbf{p}} \left\| (I_j[\mathbf{p}'] - b_j) - \frac{t_j e^{a_j}}{t_i e^{a_i}} (I_i[\mathbf{p}] - b_i) \right\|_{\gamma}$$

> Real-time (30Hz)



SVO [Forster, ICRA'14, TRO'17]

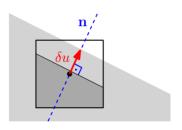
- Direct (minimizes photometric error)
 - Corners and edgelets
 - Frame-to-frame motion estimation
- Feature-based (minimizes reprojection error)
 - Frame-to-Keyframe pose refinement

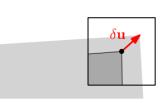


- Probabilistic depth estimation
- > SVO 2.0 includes
 - Fish-eye & Omni cameras
 - Multi-camera systems

Meant for high speed!

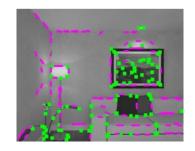
- **400 fps** on i7 laptops
- 100 fps on smartphone PC





Edgelet

Corner



Download from http://rpg.ifi.uzh.ch/svo2.html

SVO with a single camera on Euroc dataset





Processing times of SVO, LSD-SLAM, ORB-SLAM

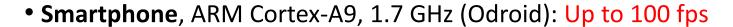
	Mean	St.D.	CPU@20 fps
SVO Mono	2.53	0.42	55 ±10%
ORB Mono SLAM (No loop closure) LSD Mono SLAM (No loop closure) DSO	29.81 23.23 20.12	5.67 5.87 4.03	187 ±32% 236 ±37% 181 ±27%

TABLE II: The first and second column report mean and standard devitation of the processing time in milliseconds on a laptop with an Intel Core i7 (2.80 GHz) processor. Since all algorithms use multi-threading, the third column reports the average CPU load when providing new images at a constant rate of 20 Hz.

[Forster, et al., SVO: Semi Direct Visual Odometry for Monocular and Multi-Camera Systems, TRO'17]

Processing Times of SVO

• Laptop (Intel i7, 2.8 GHz): up to 400 fps







Timing results on an Intel Core i7 (2.80 GHz) laptop processor:

	Thread	Intel i7 [ms]
Sparse image alignment	1	0.66
Feature alignment	1	1.04
Optimize pose & landmarks	1	0.42
Extract features	2	1.64
Update depth filters	2	1.80

Applications of SVO

Position error: 5 mm, height: 1.5 m – Down-looking camera



Robustness to dynamic scenes (down-looking camera)



Speed: 4 m/s, height: 3 m – Down-looking camera



Automatic recovery from aggressive flight [ICRA'15]

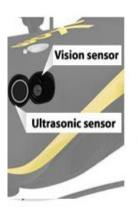


[ICRA'10-17, AURO'12, RAM'14, JFR'15, RAL'17]

Parrot: Autonomous Inspection of Bridges and Power Masts







Dacuda 3D (now Magic Leap Zurich)

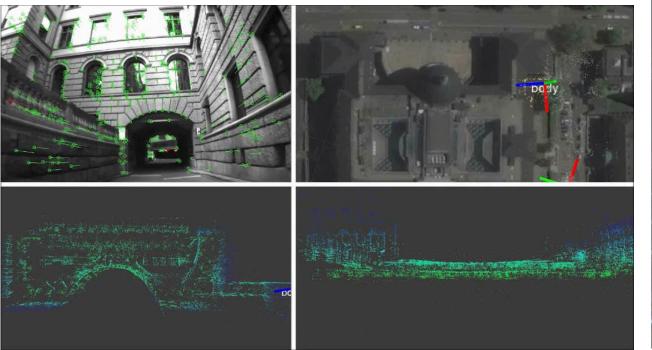
- Fully immersive VR (running on iPhone)
- Powered by SVO

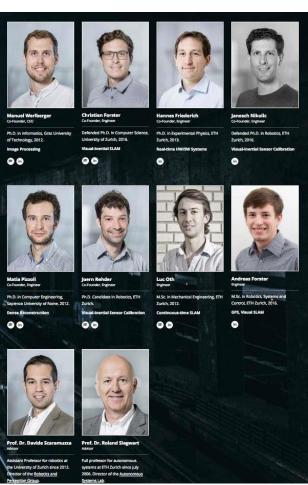




Zurich-Eye, first Wyss Zurich project, now Facebook-Oculus Zurich

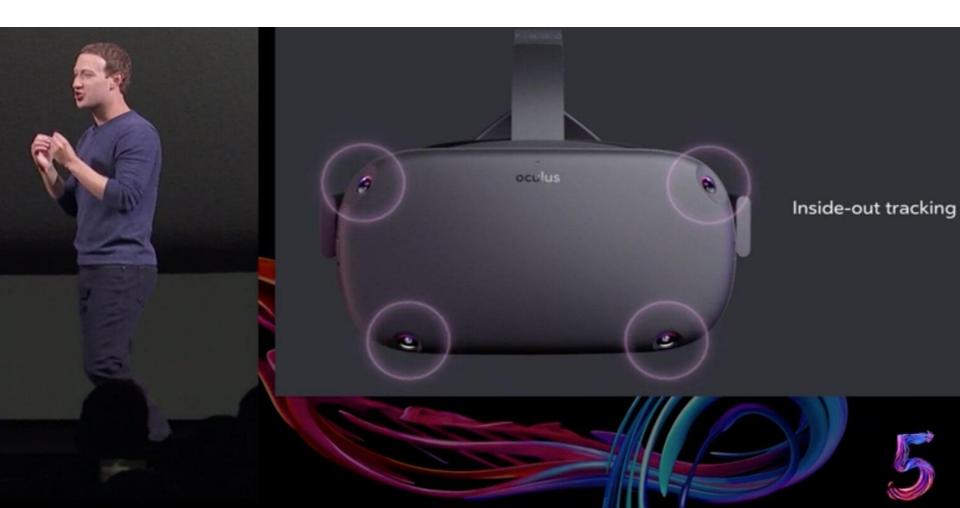
- Vision-based Localization and Mapping Solutions for Mobile Robots
- Created in Sep. 2015, became Facebook-Oculus Zurich in Sep. 2016
- > The Zurich Eye team is behind the new Oculus Quest





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Understanding Check

Are you able to answer the following questions:

- Are you able to define Bundle Adjustment (via mathematical expression and illustration)?
- Are you able to describe hierarchical and sequential SFM for monocular VO?
- What are keyframes? Why do we need them and how can we select them?
- Are you able to define loop closure detection? Why do we need loops?
- Are you able to provide a list of the most popular open source VO and VSLAM algorithms?
- Are you able to describe the differences between feature-based methods and direct methods?