Today’s Class

• Summary of the last lecture
• Camera calibration from planar grids
• Non conventional camera models
• Image filtering
Perspective Projection

Extrinsic Parameters
Radial Distortion

No distortion  Barrel distortion  Pincushion
Summary: Perspective projection equations

- To recap, a 3D world point \( P = (X_w, Y_w, Z_w) \) projects into the image point \( p = (u, v) \)

\[
\begin{bmatrix}
\tilde{u} \\
\tilde{v} \\
\tilde{w}
\end{bmatrix} = \lambda \begin{bmatrix}
u \\
v \\
1
\end{bmatrix} = K[R \mid T] \begin{bmatrix}
X_w \\
Y_w \\
Z_w \\
1
\end{bmatrix}
\]

where \( K = \begin{bmatrix}
\alpha & 0 & u_0 \\
0 & \alpha & v_0 \\
0 & 0 & 1
\end{bmatrix} \)

and \( \lambda \) is the depth \( (\lambda = Z_C) \) of the scene point

- If we want to take into account for the radial distortion, then the distorted coordinates \( (u_d, v_d) \) (in pixels) can be obtained as

\[
\begin{bmatrix}
u_d \\
v_d
\end{bmatrix} = \left(1 + k_1 r^2 \right) \begin{bmatrix}
u - u_0 \\
v - v_0
\end{bmatrix} + \begin{bmatrix}
u_0 \\
v_0
\end{bmatrix}
\]

where

\[
r^2 = (u - u_0)^2 + (v - v_0)^2
\]
Normalized image coordinates

In both computer vision and robotics, it is often convenient to use normalized image coordinates:

- Let \((u, v)\) be the pixel coordinates of an image point.
- We define the normalized image coordinates \((\bar{x}, \bar{y})\)

\[
\begin{bmatrix}
\begin{bmatrix} 
\bar{u} \\
\bar{v}
\end{bmatrix} = K^{-1}
\begin{bmatrix} 
u \\
1
\end{bmatrix}
\end{bmatrix}
\]

- Normalized image coordinates can be interpreted as image coordinates on an virtual image plane with focal length equal to 1 meter.
Camera calibration

- Calibration is the process to determine the intrinsic parameters (K) and extrinsic parameters (R, T) of the camera.
- A method proposed in 1987 by Tsai consists of measuring the 3D position of \( n \geq 6 \) control points on a three-dimensional calibration target and the 2D coordinates of their projection in the image.
- This problem is also called “Resection”, or “Perspective from n Points”, or “Camera pose from 3D-to-2D correspondence”, and is one of the most widely used algorithms in Computer Vision and Robotics.

3D position of control points is assigned in a reference frame specified by the user.
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Camera calibration from planar grids: homographies

- Tsai calibration is based on DLT algorithm, which requires points not to lie on the same plane.
- An alternative method (today’s standard camera calibration method) consists of using a planar grid (e.g., a chessboard) and a few images of this shown at different orientations.
- This method was invented by Zhang (1999).
Camera calibration from planar grids: homographies

- Our goal is to compute $K$, $R$, and $T$, that satisfy the perspective projection equation (we neglect the radial distortion)
- Since the points lie on a plane, we have $Z_w = 0$

\[
\begin{bmatrix}
\tilde{u} \\
\tilde{v} \\
\tilde{w}
\end{bmatrix} = \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \begin{bmatrix} R & T \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ 0 \\ 1 \end{bmatrix} \Rightarrow
\]

\[
\begin{bmatrix}
\alpha_u & 0 & u_0 \\
0 & \alpha_v & v_0 \\
0 & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ 0 \\ 1 \end{bmatrix}
\]

\[
\begin{bmatrix}
\tilde{u} \\
\tilde{v} \\
\tilde{w}
\end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\
0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}
\]
Camera calibration from planar grids: homographies

- Our goal is to compute K, R, and T, that satisfy the perspective projection equation (we neglect the radial distortion)
- Since the points lie on a plane, we have $Z_w = 0$

\[
\begin{bmatrix}
\tilde{u} \\
\tilde{v} \\
\tilde{w}
\end{bmatrix} =
\begin{bmatrix}
h_{11} & h_{12} & h_{13} \\
h_{21} & h_{22} & h_{23} \\
h_{31} & h_{32} & h_{33}
\end{bmatrix} \cdot
\begin{bmatrix}
X_w \\
Y_w \\
1
\end{bmatrix}
\]

\[
\Rightarrow
\begin{bmatrix}
\tilde{u} \\
\tilde{v} \\
\tilde{w}
\end{bmatrix} = H \cdot
\begin{bmatrix}
X_w \\
Y_w \\
1
\end{bmatrix}
\]

\[
\Rightarrow
\begin{bmatrix}
\tilde{u} \\
\tilde{v} \\
\tilde{w}
\end{bmatrix} =
\begin{bmatrix}
h_1^T \\
h_2^T \\
h_3^T
\end{bmatrix} \cdot
\begin{bmatrix}
X_w \\
Y_w \\
1
\end{bmatrix}
\]

where $h_i^T$ is the i-th row of $H$
Camera calibration from planar grids: homographies

\[
\begin{bmatrix}
\tilde{u} \\
\tilde{v} \\
\tilde{w}
\end{bmatrix} =
\begin{bmatrix}
h_1^T \\
h_2^T \\
h_3^T
\end{bmatrix}
\cdot
\begin{bmatrix}
X_w \\
Y_w \\
1
\end{bmatrix}
\]

Conversion back from homogeneous coordinates to pixel coordinates leads to:

\[
\begin{aligned}
u &= \frac{\tilde{u}}{\tilde{w}} &= \frac{h_1^T \cdot P}{h_3^T \cdot P} \\
v &= \frac{\tilde{v}}{\tilde{w}} &= \frac{h_2^T \cdot P}{h_3^T \cdot P}
\end{aligned}
\]

\[
\Rightarrow \\
(h_1^T - u_i h_3^T) \cdot P_i = 0 \\
(h_2^T - v_i h_3^T) \cdot P_i = 0
\]

where \( P = (X_w, Y_w, 1)^T \)
Camera calibration from planar grids: homographies

By re-arranging the terms, we obtain

\[(h_1^T - u_i h_3^T) \cdot P_i = 0 \quad \Rightarrow \quad P_i^T \cdot h_1 + 0 \cdot h_2^T - u_i P_i^T \cdot h_3^T = 0\]

\[(h_2^T - v_i h_3^T) \cdot P_i = 0 \quad \Rightarrow \quad 0 \cdot h_1^T + P_i^T \cdot h_2^T - v_i P_i^T \cdot h_3^T = 0\]

For \(n\) points, we can stack all these equations into a big matrix:

\[
\begin{pmatrix}
P_1^T & 0^T & -u_1 P_1^T \\
0^T & P_1^T & -v_1 P_1^T \\
\cdots & \cdots & \cdots \\
P_n^T & 0^T & -u_n P_n^T \\
0^T & P_n^T & -v_n P_n^T
\end{pmatrix}
\begin{pmatrix}
h_1 \\
h_2 \\
h_3
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\Rightarrow Q \cdot H = 0
\]

Q (this matrix is known)    H (this matrix is unknown)
Camera calibration from planar grids: homographies

\[ \mathbf{Q} \cdot \mathbf{H} = 0 \]

Minimal solution

- \( Q_{(n \times 9)} \) has 8 Degrees of Freedom (in fact, \( Q \) is valid up to a scale factor, thus, 9-1 = 8)
- Each point correspondence provides 2 independent equations
- Thus, a minimum of 4 non-collinear points is required

Over-determined solution

- \( n > 4 \) points
- It can be solved through Singular Value Decomposition (SVD)

Solving for K, R and T

- \( \mathbf{H} \) can be decomposed by recalling that

\[
\begin{bmatrix}
  h_{11} & h_{12} & h_{13} \\
  h_{21} & h_{22} & h_{23} \\
  h_{31} & h_{32} & h_{33}
\end{bmatrix} = \begin{bmatrix}
  \alpha_u & 0 & u_0 \\
  0 & \alpha_v & v_0 \\
  0 & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix}
  r_{11} & r_{12} & t_1 \\
  r_{21} & r_{22} & t_2 \\
  r_{31} & r_{32} & t_3
\end{bmatrix}
\]
Camera calibration from planar grids: homographies

- Demo of Camera Calibration Toolbox for Matlab (world’s standard toolbox for calibrating perspective cameras):
  http://www.vision.caltech.edu/bouguetj/calib_doc/
Application of calibration from planar grids

• Today, there thousands of application of this algorithm:
  – Augmented reality

AR Tags: http://april.eecs.umich.edu/wiki/index.php/April_Tags
Application of calibration from planar grids

• Today, there are thousands of application of this algorithm:
  – Augmented reality
  – Robotics (beacon-based localization)
• Do we need to know the metric size of the tag?
  – For Augmented Reality?
  – For Robotics?

AR Tags: http://april.eecs.umich.edu/wiki/index.php/April_Tags

RPG (us) 2013

ETH, Pollefeys group, 2010
Omnidirectional Cameras

Rome, St. Peter’s square
Overview on Omnidirectional Cameras

Omnidirectional sensors come in many varieties, but by definition must have a wide field-of-view.

- **~180° FOV**
  - Wide FOV dioptric cameras (e.g. fisheye)

- **>180° FOV**
  - Catadioptric cameras (e.g. cameras and mirror systems)

- **~360° FOV**
  - Polydioptric cameras (e.g. multiple overlapping cameras)
Catadioptric Cameras
Dioptric Cameras (fisheye)

Nikon Coolpix FC-E9 Lens
360°×183°

Canon EOS-1 Sigma Lens
360°×180°
Applications
Applications

- Xsens: IMU-fisheye-camera

(Courtesy of Xsens Technologies)
Applications

- Xsens: IMU-fisheye-camera
- Daimler, Bosch: for car driving assistance systems

(Courtesy of Daimler)
Applications

- Xsens: IMU-fisheye-camera
- Daimler, Bosch: for car driving assistance systems
- Meteorology: for sky observation
Applications

- Xsens: IMU-fisheye-camera
- Daimler, Bosch: for car driving assistance systems
- Meteorology: for sky observation
- Endoscopic Imagery: distortion removal (for the surgeon)

(Courtesy of Endo Tools Therapeutics, Brussels)
Applications

- Xsens: IMU-fisheye-camera
- Daimler, Bosch: for car driving assistance systems
- Meteorology: for sky observation
- Endoscopic Imagery: distortion removal (for the surgeon)
- RoboCup domain
Applications

- Xsens: IMU-fisheye-camera
- Daimler, Bosch: for car driving assistance systems
- Meteorology: for sky observation
- Endoscopic Imagery: distortion removal (for the surgeon)
- RoboCup domain
- Google Street View
Catadioptric cameras

- mirror
- perspective camera
Catadioptric cameras

Central catadioptric cameras

- mirror (surface of revolution of a conic)
- camera
- single effective viewpoint
Catadioptric cameras

- hyperbola + perspective camera
- parabola + orthographic lens
Why is it important that the camera be central (i.e., have a single effective viewpoint)?

- We can unwrap parts or all omnidirectional image into a perspective one.
Why is it important that the camera be central (i.e., have a single effective viewpoint)?

- We can unwrap parts or all omnidirectional image into a perspective one
- We can transform image points normalized vectors in the unit sphere
- We can apply standard algorithms valid for perspective geometry.
Omnidirectional camera calibration toolbox for Matlab (Scaramuzza, 2006)

- World’s standard toolbox for calibrating omnidirectional cameras (used at NASA, Daimler, IDS, Volkswagen, Audi, VW, Volvo, ...)

- Main applications are in robotics, endoscopy, video-surveillance, sky observation, automotive (Audi, VW, Volvo, ...)

https://sites.google.com/site/scarabotix/ocamcalib-toolbox